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Computing the closest small-signal security boundary in the control parameter space for large scale power systems

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ABSTRACT

A constrained optimization algorithm is proposed for the determination of the minimum change in the parameter values of a set of controllers that locates a complex pair of eigenvalues of a linearized power system model at a user-defined small-signal security boundary. The intended practical use is to assess the combined robustness of the system controllers in maintaining adequately damped power system oscillations. The oscillation damping security margins, given an operating point and a set of fixed damping controller parameters, is the Euclidean norm of the relative parameter variation vector, also referred in the literature as the minimum distance in the control parameter space. The computational algorithm to find the Closest Security Boundary in the Control Parameter Space, CSBCPS, is based on the rigorous mathematical implementation of the non-linear programming method to the problem, including constraints on the parameter ranges. The resulting equations are solved by the Newton method. Numerical results for a large practical power system dynamic model are detailed described to illustrate the use of the proposed CSBCPS algorithm in small-signal applications involving multiple thyristor controlled series compensators and power system stabilizers.

1. Introduction

Bifurcation methods can directly compute values of a specified set of system parameters that lead the system to the small-signal stability boundary where a system eigenvalue, either real or a complex-conjugate pair, is placed over the imaginary axis of the complex plane [1–17]. Saddle-node bifurcation methods have been applied to power flow maximum loadability studies [2–4] and Hopf bifurcation methods have been used to determine small-signal stability margins in power system oscillation studies regarding electromechanical [4–13] or subsynchronous stability dynamics [14–17].

These bifurcation methods complement the conventional small-signal stability analysis in the sense that they can determine how far the operating point is from an instability condition, showing how much the system parameters should vary to turn well-damped oscillations into undamped ones. There is a continuous interest on this topic as demonstrated by recent publications [11–13,16,17]. However, efficient and robust algorithms to calculate minimum distance to Hopf bifurcations in large scale power systems are still lacking. This paper presents an algorithm to determine the closest small-signal security boundary and the results obtained for a large practical power system model, which is general to consider any boundary and any number and type of control parameters. The two Newton algorithms in Ref. [1] compute the values of a single control system parameter that place a system eigenvalue exactly at the small-signal stability or security boundary and are especially suited to large electrical power system models. As a major extension of the work in Ref. [1], this paper proposes an optimization algorithm for the computation of the minimum distance to the security boundaries in the parameter space of the controllers, determining the minimum change in the values of a set of control parameters that would lead the linearized system to these closest small-signal stability or security boundaries.

The formulation in Ref. [1] is revisited in the next Section in order to allow a gradual and more effective description of the proposed method, presented in Section 3. Parameter range constraints are included into the algorithm in Section 4. Finally, Section 5 presents the results on the application of the proposed method to obtain the closest security margins in the control parameter space of a large-scale linearized model of the Brazilian interconnected power system.

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The proposed algorithm is the first to compute the closest small-signal security boundary considering the variation of any number and type of controller parameters associated to any equipment in large scale power system dynamic models. No similar methods have been previously presented in the literature.

2. Placing poles at the security boundary by varying a single system parameter

Consider the dynamic system modeled by a set of differential and algebraic equations (DAE) obtained by the linearization around a steady-state operating point. The parameter value that places a system pole at the small-signal stability or security boundary is computed in Ref. [1] by applying the Newton method to the following system of nonlinear equations after expanding into their real and imaginary parts.

$$\mathbf{f}(\mathbf{x}_0,p) = 0 \tag{1}$$

$$[\lambda{\cdot}\mathbf{T} - \mathbf{J}(\mathbf{x}_0,p)]{\cdot}\mathbf{v} = \mathbf{0} \tag{2}$$

$$\mathbf{c}{\cdot}\mathbf{v} = 1 \tag{3}$$

$$B(\sigma,\omega) = 0 \tag{4}$$

where $\mathbf{f}$ is a vector of functions used to calculate the initial values of the system variables defined by vector $\mathbf{x}_0$ that modifies the Jacobian matrix $\mathbf{J}$, $\mathbf{T}$ is a diagonal matrix having either ones in the lines of differential equations and zeros in the lines of algebraic equations, p is the parameter being changed to lead the system to the security or stability boundary. This boundary is determined by the locus defined by the function B, σ and ω are respectively the real and imaginary part of the system pole λ, which is the generalized eigenvalue of the linear matrix pencil ($\mathbf{J}$,$\mathbf{T}$), $\mathbf{v}$ is the generalized eigenvector of λ and $\mathbf{c}$ is a sparse line vector used for the normalization of $\mathbf{v}$.

The objective is to solve the bifurcation problem described by (1)–(4) considering the variation of a single parameter. Note that this parameter may be any system parameter or the proportional variation factor among multiple parameters.

In the general case, $\mathbf{f}$ comprises the power flow equations plus the equations related to the initialization process of the system equipment. When the parameter p that is being changed does not modify the network operating point, the power flow equations are not included. When p is a controller parameter, $\mathbf{f}$ will contain the equations for initialization of the internal variables of the controller that modify $\mathbf{J}$. If this controller is linear, $\mathbf{J}$ will not be a function of the initial values and therefore $\mathbf{f}$ will not exist.

The simplest security boundary of interest is defined by a constant damping ratio line (e.g. 5%). The following linear equation defines this security boundary for $\omega > 0$:

$$B(\sigma,\omega) = \sigma + \frac{\zeta}{\sqrt{1-\zeta^2}} \cdot \omega = 0 \tag{5}$$

where ζ is the specified damping ratio. When $\zeta = 0$, the geometric locus will coincide with the imaginary axis of the complex plane, which is the small-signal stability boundary where the Hopf bifurcations will occur.

3. Computing the closest security boundary in the parameter space

A problem of much interest is the computation of the minimum variation of a set of parameters that leads the system to the small-signal security boundary. The metric used to measure a distance in the parameter space can be, for example, the Euclidean norm of the parametric variation. Thus, the minimum norm represents the closest distance to a security boundary.

A normalized distance is considered through the division by the nominal parameter values to avoid problems with scaling. Then, the objective function to be minimized is the squared Euclidean norm of the normalized parameter distance:

$$f_{obj}(\mathbf{p}) = \sum_{i=1}^{np} \left(\frac{p_i - p_{i0}}{p_{i0}}\right)^2 \tag{6}$$

where $\mathbf{p}$ means the parameter vector of dimension np, p_i the current value for parameter $i = 1,\ldots np$ and p_{i0} the nominal value of the parameter i.

Adopting this metric for the distance is adequate when the parameters are not null or very small. Otherwise, the corresponding terms should be replaced by properly weighted absolute variations:

$$f_{obj}(\mathbf{p}) = \sum_i a_i \cdot \left(\frac{p_i - p_{i0}}{p_{i0}}\right)^2 + \sum_j a_j \cdot p_j^{\,2} \tag{7}$$

where i is the parameter index used for the relative measures and j for the absolute ones. The values a_i and a_j are the adopted weights. Any other nonlinear monotonic function whose magnitude rises with the parameter variation could be adopted as objective function, but (6) revealed an adequate choice.

The problem of finding the closest security boundary consists in minimizing this objective function subjected to the constraint that a complex pair of poles will lie at the boundary B. This is an optimization problem, more specifically, a nonlinear programming (NLP) problem [18,19]. In (9)–(13) the proposed NLP equations are presented.

$$\mathrm{Min}\, f_{obj}(\mathbf{p}) \tag{9}$$

$$\mathrm{S.t.:}\ \mathbf{f}(\mathbf{x}_0, \mathbf{p}) = 0 \tag{10}$$

$$[\lambda{\cdot}\mathbf{T} - \mathbf{J}(\mathbf{x}_0, \mathbf{p})]{\cdot}\mathbf{v} = \mathbf{0} \tag{11}$$

$$\mathbf{c}{\cdot}\mathbf{v} = 1 \tag{12}$$

$$B(\sigma,\omega) = 0 \tag{13}$$

The equations that define the constraints are the same ones seen at Section 2, considering, however, that now a vector of parameters $\mathbf{p}$ is varied rather than only one parameter p. The proposed algorithm to solve iteratively this problem by varying a set of controller parameters is here denominated Closest Security Boundary in the Control Parameter Space (CSBCPS).

The constraints described by complex equations should be converted into real equations, by expanding them into rectangular coordinates:

$$\mathbf{f}(\mathbf{x}_0, \mathbf{p}) = \mathbf{0} \tag{14}$$

$$[\sigma{\cdot}\mathbf{T} - \mathbf{J}(\mathbf{x}_0, \mathbf{p})]\mathbf{v}_{\mathrm{Re}} - [\omega{\cdot}\mathbf{T}]\mathbf{v}_{\mathrm{Im}} = \mathbf{0} \tag{15}$$

$$[\omega{\cdot}\mathbf{T}]\mathbf{v}_{\mathrm{Re}} + [\sigma{\cdot}\mathbf{T} - \mathbf{J}(\mathbf{x}_0, \mathbf{p})]\mathbf{v}_{\mathrm{Im}} = \mathbf{0} \tag{16}$$

$$\mathbf{c}\,\mathbf{v}_{\mathrm{Re}} = 1 \tag{17}$$

$$\mathbf{c}\,\mathbf{v}_{\mathrm{Im}} = 0 \tag{18}$$

$$B(\sigma,\omega) = 0 \tag{19}$$

This NLP problem can be written in a compact form as:

$$\mathrm{Min}\, f_{obj}(\mathbf{p}) \tag{20}$$

$$\mathrm{S.t.:}\ \mathbf{h}(\mathbf{x}, \mathbf{p}) = 0 \tag{21}$$

where vector $\mathbf{x}$ is a vector of dimension nx comprised of the variables $\mathbf{x}_0$, $\mathbf{v}_{\mathrm{Re}}$, $\mathbf{v}_{\mathrm{Im}}$, σ, ω and vector $\mathbf{h}$ is comprised of the constraints given in (14)–(19) of dimension nh.