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Distributionally robust optimization for energy and reserve toward a low-carbon electricity market

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ABSTRACT

This paper proposes a two-stage distributionally robust model for the optimization of energy and reserve under uncertain wind power. The first-stage model considers a day-ahead market that determines the nominal generation and reserves before the realization of wind power uncertainty. The second-stage decisions are made in a realtime market, after the observation of uncertainty, so that the expected emission factor is constrained below a target level. Case studies are conducted to demonstrate that the proposed method is capable of effectively capturing the ambiguous distribution of wind power generation, and can be tractably solved. The influence of different emission constraints is also discussed, showing the trade-off between lowering the total operating cost and reducing the long-term impact of carbon emissions.

1. Nomenclature

We use bold letters to denote vectors or matrices. Entries of vectors or matrices are regular letters with the corresponding subscripts. For example, b_i is the ith entry of vector $\boldsymbol{b}$, and A_{mn} is the entry of matrix $\boldsymbol{A}$ in the mth row and the nth column. The nth column of $\boldsymbol{A}$ is denoted by $\boldsymbol{A}_n$. $|\mathcal{S}|$ is the number of elements in set $\mathcal{S}$, and $\|\cdot\|$ is the 2-norm of a vector. The currency unit used in this paper is US dollars ($). Other notations are listed below.

1.1. Indices and sets

$a/\mathcal{A}$	Indices/set of conventional thermal units
$b/\mathcal{B}$	Indices/set of load buses
$h/\mathcal{H}$	Indices/set of energy storage systems
$i/\mathcal{I}$	Indices/set of random variables
$j/\mathcal{J}$	Indices/set of auxiliary variables
$k/\mathcal{K}$	Indices/set of constraints in the extended support set
$l/\mathcal{L}$	Indices/set of transmission lines
$m/\mathcal{M}$	Indices/set of uncertain constraints
$\mathcal{P}_0(\cdot)$	Set of distributions of given random variables
$s/\mathcal{S}$	Indices/set of wind power sources
$t/\mathcal{T}$	Indices/set of time steps
$\mathcal{V}$	Feasible set of unit commitment decisions
τ	Indices of time steps

1.2. Uncertainty model

$\mathbb{F}$	Ambiguity set of random variables
$\mathbb{G}$	Extended ambiguity set
$\mathbb{P}$	Distribution of random variables $\tilde{\boldsymbol{z}}$
$\mathbb{Q}$	Joint distribution of vectors $\tilde{\boldsymbol{z}}$ and $\tilde{\boldsymbol{u}}$
$\tilde{\boldsymbol{u}}$	Auxiliary variables introduced into the extended sets
$\hat{w}_{st}$	Expected wind power from source s at time t (MW)
$\tilde{z}_{st}$	Forecast error of wind power source s at time t (MW)
$\mathcal{Z}/\hat{\mathcal{Z}}$	Support/extended support set of random variables

1.3. Constants and functions

C_a^d/C_a^u	Cost of downward/upward reserves of unit a ($/MW)
C_a^s	Cost of starting up unit a ($)
E_h	Energy rating capacity of storage system h (MWh)
F^e	Target of the expected emission factor (kg/MWh)
$F_l(\cdot)$	Function of the DC power flow for the lth line
L_{bt}	Load at bus b, during time step t (MW)
M_0	Number of rows in matrix $\boldsymbol{G}$
M_k	Number of rows in matrix $\boldsymbol{A}_k$
N_1	Number of decisions for the day-ahead market
$\bar{P}_a$	The maximum capacity of thermal unit a (MW)
$\underline{P}_a$	The minimum capacity of thermal unit a (MW)

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Q_h Power rating capacity of storage system h (MW)
R_a^d Ramp-down rate limitation of thermal unit a (MW/h)
R_a^u Ramp-up rate limitation of thermal unit a (MW/h)
T_l Transmission capacity of the lth line (MW)
$\bar{Z}_{st}$ Upper bound of forecast error $\tilde{z}_{st}$ (MW)
$\underline{Z}_{st}$ Lower bound of forecast error $\tilde{z}_{st}$ (MW)
α_a Squared term of the cost function of unit a (\$/MWh^2)
β_a Linear term of the cost function of unit a (\$/MWh)
γ_a Constant term of the cost function of unit a (\$)
δ_h^c Charge efficiency coefficient of storage system h
δ_h^d Discharge efficiency coefficient of storage system h
ϵ_a Carbon emission rate of unit a (kg/MWh)
θ_t Constant indicating the skewness of wind power distribution at time t
ϕ_{1t} The mean absolute deviation of the forecast error of total wind power at time t (MW)
ϕ_{2t} The standard deviation of the forecast error of total wind power at time t (MW)

1.4. Daily-ahead market decisions

d_{ht}^0 Nominal discharge of energy storage h at time t (MW)
o_{at} Start-up cost of thermal unit a at time t (\$)
p_{at}^0 Nominal output of thermal unit a at time t (MW)
q_{ht}^0 Nominal charge of energy storage h at time t (MW)
r_{at}^d Downward reserve of thermal unit a at time t (MW)
r_{at}^d Upward reserve of thermal unit a at time t (MW)
v_{at} Unit commitment decision of unit a at time t
w_{st}^0 Nominal output of wind power source s at time t (MW)
$\boldsymbol{x}$ Vector of all daily-ahead market decision variables

1.5. Realtime market decisions and decision rule

$d_{ht}(\boldsymbol{z})$ Discharge of storage h at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$p_{at}(\boldsymbol{z})$ Output of thermal unit a at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$q_{ht}(\boldsymbol{z})$ Charge of storage h at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$w_{st}(\boldsymbol{z})$ Output of wind farm s at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$\boldsymbol{y}(\boldsymbol{z})$ Vector of all realtime market decisions, under uncertainty realization $\boldsymbol{z}$
$\bar{\boldsymbol{y}}(\boldsymbol{z}, \boldsymbol{u})$ Decision rule as affine functions of random variables $\boldsymbol{z}$ and auxiliary variables $\boldsymbol{u}$

2. Introduction

Global warming has become a serious issue in the 21st century [1–3], and there is a pressing need to reduce carbon emissions in power industry. In an effort to achieve low-carbon electricity markets, clean energy technologies are applied in fast-growing scales in modern power systems. Typical clean energy sources, such as wind and photovoltaic power, are known to be highly uncertain and difficult to dispatch. This is why various optimization approaches are studied these years to model highly volatile uncertain energy sources in joint energy and reserve optimization.

For example, stochastic programming is widely used to achieve the optimal expected performance [4–8], where the uncertainty of renewables is represented by a number of scenarios. Such a scenario-representation, however, requires detailed information on the exact probability distribution of random variables [9], which may be difficult to be accurately identified. Even if the detailed distribution information is available, the number of scenarios might grow exponentially with the increase of random parameters [10]. Though various decomposition algorithms [11–13] are developed to alleviate the computational burden, the stochastic programming problems remain very challenging to solve. Besides stochastic programming, chance-constrained programming [14] also greatly replies on the precise information on probability distributions. Chance constraints are generally non-convex and intractable to solve, except for special cases like having Gaussian distributed random variables [15].

Robust optimization seeks optimal solutions that are robust against the worst-case realizations over a deterministic uncertainty set [16], so it can be applied to energy and reserve optimization problems [17–20] without assuming the actual probability distribution of uncertain parameters. Similar to robust models, the interval optimization is frequently used to protect the system against the worst-case scenarios defined by the boundaries of uncertain renewable generation [21–23]. However, both robust and interval optimization approaches are unable to directly model expected terms, because limited distribution information can be explicitly incorporated into the uncertainty set or the uncertainty boundaries.

In order to address these difficulties, a new method called distributionally robust optimization [24–26] has been introduced to power system optimization [27–29]. This method characterizes the system uncertainties by some descriptive statistics rather than detailed distribution information, so that the worst-case expectation expressions can be formulated in the objective function or in constraints without enumerating scenarios.

This paper proposes a distributionally robust model for the day-ahead scheduling of energy and reserve considering uncertain wind power generation. Constraints on the expected emission factor of all generators, which cannot be directly modeled by conventional robust or interval optimization approaches, are imposed to control the long-term impact of carbon emissions. Instead of relying on the knowledge of the exact probability distribution, the proposed method captures the uncertainty of wind power generation by an ambiguity set containing a collection of distributions. Compared with the other distributionally robust models [27,28] that merely depends on the mean values and covariance to define the ambiguity set, our method imposes a finite support set and uses additional distribution information, such as mean absolute deviations and the asymmetry of distribution functions, to better describe the possible pattern of distributions, thus improving the quality of solutions. These statistical measures can be more easily evaluated by using point forecast [30–34] and prediction interval approaches [35–37]. For the rest of the paper, the proposed formulation and details of deriving a tractable robust counterpart are presented in the next section. Case studies are provided in Section 4, and the final section concludes our work.

3. Formulation

3.1. Uncertainty model of wind power

In this paper, the uncertain wind power is expressed as Eq. (1).

$$\tilde{W}_{st} = \hat{w}_{st} + \tilde{z}_{st}, \quad \forall s \in \mathcal{S}, \quad \forall t \in \mathcal{T} \tag{1}$$

where $\hat{w}_{st}$ denotes the expected power of wind energy source s at time step t, determined by any forecast technologies [32–34], and $\tilde{z}_{st}$ is a random variable indicating the corresponding forecast error of wind power.