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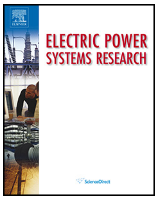

Voltage modelling of LV feeders with dispersed generation: Probabilistic analytical approach using Beta PDF

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ABSTRACT

The published probabilistic analytical modelling of voltages on a low voltage (400/230 V) feeder with passive loads is extended and modified to calculate the variation with dispersed generation (DG). The existing method was based on representing residential loads as Beta distributed currents at any interval of demand, usually the maximum demand or the interval with maximum voltage drop. Now, in addition to probabilistic loads modelled as positive currents out of a feeder, DG is included by modelling the injection as negative current variables described by their Beta PDFs. The statistical elements are manipulated by keeping the load and generation notionally separate at each node so that superposition can be applied to assess separately their effects. This analytical approach calculates the probable voltage drop or rise on the feeder for a selected level of probabilistic confidence. The proposed approach is compared with Monte Carlo simulations. All steps required to perform the calculations are linear and fast because they require no iterations.

1. Introduction

In South Africa, and elsewhere, low voltage (400/230 V) single-phase, bi-phase or three-phase feeders supply single phase residential and small non-residential loads, and systems in other countries supply similar loads at 110 V (phase-neutral). All these loads may be characterised as currents at nominal voltage, rather than power, and represented statistically as Beta probability density functions (PDFs) [1–3]. The parameters of a Beta PDF are the shape parameters α, β and a scaling factor C. These parameters are readily extracted from the large volume of 5-minute averaged residential load data collected in South Africa, and they have been used in LV distribution design since 1999 [4]. Exploiting the characteristics of the Beta PDF, a probabilistic (statistical analytical) method of voltage calculation on radial feeders was developed [3–5]. It is essentially a transform of the current PDF to a voltage drop PDF and is known as the Herman Beta method in a South African National Standards document [4] and will be referred to as the HB transform.

In a passive feeder, with only loads, the main design need is to evaluate the expected voltage drop on the feeder at the time of maximum demand (usually during winter in South Africa). Since the HB transform is probabilistic, a range of output voltage drops is calculated as an output Beta PDF, from which a single decision-making value is extracted that corresponds to a level of statistical confidence or, conversely, a level of risk. A typical value for this risk is 10% [4], which is equivalent to the 90-percentile value of the resultant voltage drop PDF, but the level of risk adopted also depends on the details of the derivation of the load model and the application of the calculations [6].

In recent years the argument for accommodating renewable energy sources in distribution feeders has gained momentum. In a feeder with dispersed or distributed generation (DG), an additional concern is that the voltage rise may exceed regulatory limits. This voltage rise does not occur at the interval of maximum demand (assumed as the basis of design for passive feeders), but rather when the currents from the DG are high and the passive load currents on the feeder are low. In the case of photovoltaic (PV) generation, for example, this may occur at around midday in summer, unless there is high air-conditioning load. Therefore, the characteristic load data for active feeders should include the community's low load periods as well as the maximum demand.

Including the variability of both loads and generation is a problem. Given the asymmetrical variability of the load demand, a simple deterministic approach could give misleading results. There are two possible ways of dealing with the problem. Monte Carlo simulation (MCS) is slow [7,8], because for each feeder configuration it draws a large number of random load current samples from

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a set of measurements or a statistical description (PDF) of measured values, and repeats this many times. An alternative analytical approach [9] is based on the statistical moments of the Gaussian distribution of the loads, although that PDF was found to be inadequate in the South African load research experience [1] because the load current distributions are skewed. Another analytical approach is to derive the output PDF by the method of combined cumulants together with a Gram-Charlier Expansion [10], reported by Zhang [11] to be significantly faster than the MCS approach. The accuracy of the method appears to rely on the order of the moments used (3rd–9th) and it is unclear how the order should be chosen. In the examples presented [10,11] the loads were modelled by their means and variances, without representing asymmetry, and are applied to balanced three-phase systems. By contrast, the analytic-probabilistic approach taken in the HB transform includes skewed load PDFs and the effects of unbalanced connections.

Based on the key capabilities of the Beta PDF to represent skewed, positive distributions on a finite base consistent with actual loads on distribution systems and the outputs of DG and, therefore, the practical usefulness of the model, Gaunt experimented with modelling DG as negative Beta loads modelled as Beta PDFs in the HB transform, and the preliminary findings were presented in a tutorial at a conference in 2011 [12]. The modified HB transform with loads and DG is still a transform without iterations, resulting in a probabilistically determined envelope of voltage drops and voltage rises according to the risk level adopted. However, the way risk is specified for active feeders differs from that for passive feeders.

The development of the modified HB transform is achieved through three properties: the representation of DG as negative load current, the separation of loads and DG at a node by adopting sub-nodes to retain algebraic integrity and the power of superposition, and the detailed modification of the HB transform to enable the calculation of the effects of a generator instead of a load with the correct probabilistic confidence. The modifications are presented in four main sections: a description of the structure of models for feeder analysis; changes needed to an already-published algorithm; testing the modified algorithm; and an example and conclusions.

2. System modelling

The analysis of an active LV feeder system requires models of the load, DG and feeder, and the transform of the input parameters into a result.

2.1. Load model

Conventional voltage magnitude performance assessment for LV feeders evolved from approaches applicable to MV power systems. At the LV feeder level, however, there are fewer connected loads on a typical feeder and the load model must represent both the magnitude and the significant variability with time and between the individual loads. Fig. 1 shows mean load current profiles using 5-minute intervals for a typical winter and summer day of an urban residential South African household typical of a socio-economic group likely to install PV-DG, as extracted from the NRS Load Survey of many individual households in each of many different communities. (The sampling interval is determined by the Nyquist criterion for sampling consistent with the 10-minute periods specified for most power quality assessments of voltage magnitude.) In addition, Fig. 1 also shows a typical hourly current output profile of a nominal 2 kW solar PV installation at a household. Two time intervals on the profiles show the feeder's peak load at interval A and the minimum load during sunlight at B.

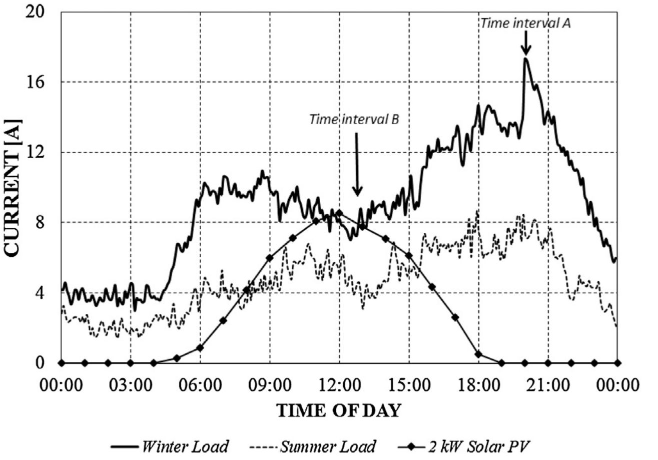

Fig. 1. Typical winter and summer load profiles, for an average urban residential household in a community in South Africa, with a PV profile for the same location.

Table 1
Parameters of the distribution of load currents at intervals A and B of Fig. 1.

Load Demand	Interval	Mean [A]	Std Dev. [A]	Beta PDF parameters			Power [kVA]
				α	β	C [A]	
Winter	A	17.46	10.50	1.67	4.07	60	4.01
	B	7.07	8.12	0.55	4.12	60	1.63
Summer	B	3.71	4.49	0.580	8.798	60	0.85

Table 2
Parameters of the distribution for maximum output for a nominal 1 kW PV DG with 80% efficiency.

Power output [kW]	Mean [A]	Std Dev [A]	Alpha, α	Beta, β	Scaling, C [A]
0.8	3.478	0.154	255.50	255.50	6.957

Table 1 presents the Beta shape parameters α and β on a base of C = 60 A, derived from the mean and standard deviation of the measured currents, for three intervals on the load profiles of Fig. 1. Clearly, in studies of voltage drop or voltage rise at different times of day or season, the representative parameters of the load are needed.

2.2. PV DG model

Many DG sources are subject to weather conditions. The output from PV panels is obviously only available between sunrise and sunset, so it has a daily profile dependant on the season. PV output during any interval may be expressed as a PDF and various authors [8,13,14] have suggested that a Beta PDF is appropriate for this purpose.

The approach of representing PV-DG on a feeder as a negative load, injecting power into the feeder, has been used by others [15,16], although with constant values of P and Q. For use with the HB transform, the model is a Beta PDF of the injected PV currents, and represents the variability between different PV installations. Measured solar data tends to result in a PDF with broad dispersion arising from the shadowing, orientation, tilt and efficiency of individual installations. If extreme voltage rise conditions are to be considered, the generation can be modelled as a narrowly dispersed PDF equivalent to the maximum PV output from well-oriented PV panels producing their maximum output current at interval B. Table 2 shows the parameters to model maximum PV current from 1 kW-rated solar panels, assuming 80% efficiency, and with high shape parameters producing a spike-like PDF. In practical