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Reduced-order equivalent model to large power networks derived from its spectral dispersion

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ABSTRACT

This paper describes a general methodology for identification of a reduced-order dynamic equivalent with modal frequency distribution to large power networks, derived from its frequency-varying response. The method is used to define a state space model with modal frequency dispersion established from both, the application of the empirical orthogonal functions (EOFs) analysis and vector fitting (VF) procedure for rational functions approximation from frequency-domain data sets. Initially, our approach uses orthogonal modes of major contributions of spectral dispersion derived from the EOFs analysis to construct a reduced-order approximation with applications to multiple-input, multiple-output (MIMO) linear-time invariant (LTI) systems. This approximation defines an optimal distributed solution to the frequency-varying data set, where their fundamental properties are based on the interpretation of pre-selected frequencies contained into the eigenvectors of a cross-spectrum matrix. Once the reduced-order empirical modal decomposition is derived, its coefficients are used in the VF procedure in order to generate a rational function approximation into a frequency band with particular level of kinetic energy with applications to MIMO systems. Additionally, the reduced-order equivalent network in a state space model is derived from a VF, which can be efficiently incorporated in a power network simulator to electromechanical studies of multimachine dynamics with modal frequency splitting. Finally, an example for large power networks is examined to both demonstrate the effectiveness for fitting reduced-order dynamic equivalents and to capture its modal coherence and frequency distribution.

1. Introduction

Over the last years, many techniques using linear and nonlinear fitting routines have been proposed to fit the frequency response of large power networks. This is in order to ensure accurate equivalent models and decrease remarkably the computational burden in simulators of transient studies where the fitting may be performed either in the s or z domain [1–9]. The analysis of frequency range considering accuracy, the shape of the frequency response, the mathematical model and the possibility for time-domain implementation are examined to decide which one fitting technique is the most appropriated. A recent problem related with these methodologies has been the derivation of a reduced-order equivalent model with applications to multiple-input multiple-output (MIMO) linear-time invariant (LTI) systems, that considers the modal-geographical dispersion of large interconnected power networks and its distributed implementation to multimachine systems. This represents a drawback to be efficiently incorporated into a power systems simulator to electromechanical studies [7,10–19] and it will be treated in this work. In [19] is presented an algorithm for identifying a multiphase network equivalent for transient simulations of single-input single-output (SISO) systems, where the method is limited to use the trace of the associated transfer function matrices and to divide the network in two parts: a study zone and a external zone, which the computational efficiency with acceptable degree of accuracy from the results are derived. Additionally, in [2] is given a general methodology to the order reduction of dynamic models by using the singular value decomposition and balanced realization techniques in SISO systems where the issues of sparsity, convergence, and accuracy are examined. Recently, a statistical identification method established on the basis of the empirical orthogonal functions (EOFs) analysis, more commonly called principal components (PCs) analysis, has been widely applied to identify and to extract modal instabilities from a data set. The technique is based on the correlation structure from time-varying fields,

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which can treat both optimal modal distribution and geographical-spatial dispersion [20–24]. This fact motivates us to extend the EOFs analysis to the studies of frequency-domain responses of large power networks and approaching a reduced-order equivalent model with applications to MIMO systems from a data set. Therefore, in this paper a general methodology for identification of a reduced-order equivalent network with modal frequency distribution to power networks derived from its frequency-domain response, is presented. The method is used to define a state space model with modal frequency splitting established on the application of the EOFs analysis and the vector fitting (VF) procedure for rational functions approximation from frequency-domain data. On the one hand, our approach uses the orthogonal modes of major contributions of modal coherence derived from the EOFs analysis to construct a reduced-order approximation. This approximation defines an optimal distributed solution to the frequency-varying data set. The fundamental properties of this data set are based on the interpretation of pre-selected frequencies contained into the eigenvectors of a cross-spectrum matrix. Once the reduced-order empirical modal decomposition is derived, its spectral coefficients are used in the VF procedure in order to generate a rational function approximation into a particular frequency band with particular level of kinetic energy of applications to MIMO systems. On the other hand, the reduced-order equivalent network given in a state space model form is derived from the VF and efficiently incorporated in a power network simulator to electromechanical studies of multimachine dynamics and inter-area control system design. The procedure incorporates frequency domain responses to study and to characterize coupling frequencies and geographical dispersion into multiple power networks used to the analysis of its oscillatory activity. In addition, the method also incorporates a procedure based in frequency band to effectively define a reduced model from the data used. The objective of this study is to infer the relationship between modal frequencies and the spatial relationship of waveforms present within a particular frequency interval. Some of these difficulties are discussed in the interpretation of results, where a power network model is examined to demonstrate the effectiveness when fitting reduced-order equivalent networks capturing its modal coherence and distribution.

This paper is organized as follows: Section 2 introduces some theoretical backgrounds about the EOFs analysis; the method of VF is described in Section 3; next, the proposed method is presented in Section 4; test results are provided in Section 5; finally, discussions and conclusions are given in Sections 6 and 7, respectively.

2. Empirical orthogonal functions analysis

The EOFs analysis is developed to be applied for representations of a data set $\mathbf{X}(x_j, t_k) \in \mathbb{R}^{m\times n}$, with $n \ll m$, where x_j to $j = 1, \ldots, n$, represents the spatial variables and t_k with $k = 1, \ldots, m$, is the time at which the observations are made. A statistical decomposition illustrates the phenomenon of modal distribution, which is derived from the response of large interconnected systems. In [22–24], the model based on the EOFs analysis is given by:

$$\mathbf{X}(x,t) \in \mathbb{R}^{m\times n} = \mathbf{X}_{\text{swc}}(x,t) + \mathbf{X}_{\text{twc}}(x,t), \tag{1}$$

where $\mathbf{X}_{\text{swc}}$ and $\mathbf{X}_{\text{twc}}$ denote standing and traveling waveform components, respectively. The method is established into split a complex autocorrelation matrix computed from the resulting data array defined as:

$$\mathbf{C} \in \mathbb{C}^{n\times n} = \frac{1}{m}\mathbf{X}_C^H(x,t)\mathbf{X}_C(x,t) = \mathbf{C}_R + i\mathbf{C}_I, \tag{2}$$

with $i = \sqrt{-1}$; where the subscripts C, R and I indicate complex, real and imaginary vectors, respectively, while the superscript H denotes the conjugate transposed of a complex matrix. Implicit in the model is the assumption that $\mathbf{X}(x, t)$ is augmented by their imaginary components to form a complex data matrix, $\mathbf{X}_C(x, t)$, which can be represented as:

$$\mathbf{X}_C(x,t) = \|\mathbf{X}_C\|[\cos(\boldsymbol{\theta}_{X_C}t) + i\ \ \sin(\boldsymbol{\theta}_{X_C}t)], \tag{3}$$

where $\|\mathbf{X}_C\|$ and $\boldsymbol{\theta}_{X_C}$ are the magnitude and phase of $\mathbf{X}_C$, respectively. Under this assumption, it can be easily verified that:

$$\mathbf{C}_R = \|\mathbf{X}_C^\top\|\mathbf{X}_C\|[\cos(\boldsymbol{\theta}_{X_C^\top}t)\cos(\boldsymbol{\theta}_{X_C}t) + \sin(\boldsymbol{\theta}_{X_C^\top}t)\sin(\boldsymbol{\theta}_{X_C}t)], \tag{4}$$

and

$$\mathbf{C}_I = \|\mathbf{X}_C^\top\|\mathbf{X}_C\|[\cos(\boldsymbol{\theta}_{X_C^\top}t)\sin(\boldsymbol{\theta}_{X_C}t) - \sin(\boldsymbol{\theta}_{X_C^\top}t)\cos(\boldsymbol{\theta}_{X_C}t)], \tag{5}$$

where can be seen that when the time is in phase with the extremum of the cosine or sine, the resulting matrix $\mathbf{C} = \mathbf{C}_R + i\mathbf{C}_I$ from (2) is a Hermitian matrix, where its real part is a symmetric matrix, *i.e.*, $\mathbf{C}_R = \mathbf{C}_R^\top$, where the superscript $\top$ indicates transposed vector, and $\mathbf{C}_I$ is an asymmetric matrix or hemisymmetric matrix, *i.e.*, $\mathbf{C}_I^\top = -\mathbf{C}_I$, with $\det(\mathbf{C}_I^\top) = 0$ when its size is odd. Since the symmetrical matrix is a particular case of the Hermitian matrix, then its eigenvectors are real with eigenvalue $\lambda_1 > \lambda_2 \cdots > 0$. Due to the fact that all of the elements for the asymmetrical matrix are purely imaginary, then it is a normal matrix, *i.e.*, all of its eigenvectors are in the complex conjugate form. Therefore, the optimal orthogonal basis for the modal decomposition is defined by eigenfunctions $\boldsymbol{\varphi}_R(x)$ and $\boldsymbol{\varphi}_I(x)$ for both real and imaginary parts of (2). The orthogonal basis defined in the infinite-dimensional Hilbert space $\boldsymbol{L}^2([0, 1])$ are computed from the eigenvalue problem by solving the linear system with the form $\mathbf{C}\boldsymbol{\varphi}(x) = \boldsymbol{\lambda}\boldsymbol{\varphi}(x)$, and it is optimal in the sense that maximizes the average projection of the response matrix $\mathbf{X}(x, t)$, suitably normalized:

$$\max_{\langle\varphi\rangle\in L^2([1,0])} \langle|\,(\mathbf{X}(x,t), \langle\boldsymbol{\varphi}(x)\rangle)|^2\rangle \quad \text{subject to} \quad \|\ \langle\boldsymbol{\varphi}(x)\rangle\ \|^2 = 1, \tag{6}$$

where $|\cdot|$ denotes the modulus, $||\cdot||$ is the L^2-norm and $\langle\cdot\rangle$ implies the use of an average operation. Then, the associated approximation to the data set in terms of a truncated sum of dominant empirical modes p and q derived from the kinetic energy contribution contained in the eigenvalue $\lambda_1 > \lambda_2 \cdots > 0$, is given by:

$$\mathbf{X}(x,t) \in \mathbb{R}^{m\times n} = \text{real}\left[\sum_{j=1}^{p}\boldsymbol{a}_{R(j)}(t)\varphi^*_{R(j)}(x) + i\sum_{j=1}^{q}\boldsymbol{a}_{I(j)}(t)\varphi^*_{I(j)}(x)\right], \tag{7}$$

where * denotes the complex conjugation with temporal coefficients $\boldsymbol{a}(t) \in \mathbb{C}$, which are computed as:

$$\boldsymbol{a}_j(t) = \langle\mathbf{X}(x,t), \varphi_j(x)\rangle / \langle\varphi_j(x), \varphi_j(x)\rangle. \tag{8}$$

From (7), it should be noticed that the statistical representation given from the EOFs analysis is established through the coefficients $\boldsymbol{\varphi}(x)$ and $\boldsymbol{a}(t)$, which represent an optimal modal distribution to the data set. In $\boldsymbol{\varphi}(x)$ are given both, the spatial modal distribution and the variability associated with the mode shape, while $\boldsymbol{a}(t)$ is used to study the modal geographical dispersion. Moreover, the phase variability shows the relative phase fluctuation among various spatial locations where the modal distribution is defined. The modal distribution amplitude and its phase variability are computed, respectively, from (7) by:

$$S = \sqrt{\boldsymbol{\varphi}^*(x)\boldsymbol{\varphi}(x)}, \tag{9}$$

$$\boldsymbol{\Phi} = \tan^{-1}\left(\frac{\boldsymbol{\varphi}_I(x)}{\boldsymbol{\varphi}_R(x)}\right). \tag{10}$$

Moreover, a measurement of the modal variability, in magnitude and phase, for a particular oscillation defined into the spatial modal structure is given by $\boldsymbol{a}(t)$. This information is usually used in