Electric Power Systems Research 143 (2017) 389–399

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Iterative infinitesimal generator discretization-based method for eigen-analysis of large delayed cyber-physical power system

Hua Ye*, Weikang Gao, Qianying Mou, Yutian Liu

Key Laboratory of Power System Intelligent Dispatch and Control of Ministry of Education (Shandong University), 17923 Jingshi Road, Ji'nan 250061, China

ARTICLE INFO

Article history:
Received 7 April 2016
Received in revised form
12 September 2016
Accepted 9 October 2016
Available online 7 November 2016

Keywords:
Cyber-physical power system
Eigen-analysis
Infinitesimal generator discretization
Small signal stability
Time delay
Wide-area damping control
Wide-area measurement system

ABSTRACT

To efficiently analyze the stability of large delayed cyber-physical power systems (DCPPS) incorporating wide-area damping controllers, an iterative infinitesimal generator discretization-based method (IIGD) for computing critical eigenvalues of the system is presented. IIGD contains three core techniques to guarantee efficiency and scalability. First, the sparsity of the infinitesimal generator's discretized matrix, which possesses identical spectrum to DCPPS, is explored by reformulating its blocks into Kronecker products. Especially, the dominant block is factorized as sum of Kronecker products of constant Lagrange vectors and system state matrices, which lays the basis of further utilizing the sparsities in the augmented state matrices of DCPPS. Second, the shift-invert preconditioning technique is applied to transform the required eigenvalues into those dominated in moduli. Third, the inverse iteration of the discretized matrix involved in sparse eigenvalue computation is iteratively achieved by utilizing the induced dimension reduction method (IDR(s)). Subsequently, the discretized matrix–vector product embedded in the method is efficiently implemented by exploiting the unique property of Kronecker product and the inherent sparsities in augmented system state matrices. The correctness, accuracy, efficiency and scalability of IIGD are extensively studied and thoroughly validated on the two-area four-machine test system and a real-life large transmission grid.

1. Introduction

The real-life power grids with wide-area damping controllers embedded are large delayed cyber-physical power system (DCPPS) in nature [1], because time delays up to several hundreds of milliseconds are introduced during processing and transmitting wide-area measurements [2,3]. Therefore, it is critical and desirable for small signal stability analysis methods with the ability of dealing with large DCPPS, i.e., scalability, so that the time delay impacts can be accurately evaluated. With this aim, an iterative infinitesimal generator discretization (IIGD)-based eigenvalue computation method is presented in the paper.

1.1. Literature review

Existing studies on small-signal stability of DCPPS can be generally divided into two categories: time-domain methods and frequency-domain methods.

In the time domain, Lyapunov–Krasovskii functional-based delay-dependent stability criteria have been proposed for analyzing the asymptotic stability of DCPPS. Especially, the maximum time delay that a power system can tolerate and remain stable, i.e., delay margin, can be obtained by solving a series of generalized eigenvalue minimization problems with linear matrix inequality constraints [4]. However, the time-domain methods are sufficient conditions for asymptotic stability and substantially conservative. Their accuracies are further compromised since model reduction is always accompanied to reduce the cumbersome computation burden. In addition, only a few studies in the time domain, e.g., [5,6], can adapt to multiple time delays.

In the frequency domain, delays are naturally transformed into exponential terms. Correspondingly, the characteristic equation of DCPPS becomes transcendental and has an infinite number of eigenvalues [7]. For ease of stability analysis and control synthesis, lots of efforts are made on reducing the infinite-dimensional eigenvalue problem of DCPPS into a finite-dimensional one. Subsequently, critical (such as rightmost) eigenvalues of the system can be accurately computed. According to different ways of reducing the eigen-problem, the frequency-domain methods can be further classified as *delay substitution/approximation-based methods* [8,9] and *spectral discretization-based methods* [10].

* Corresponding author.
E-mail address: yehua@sdu.edu.cn (H. Ye).

http://dx.doi.org/10.1016/j.epsr.2016.10.016
0378-7796/

Delay substitution/approximation-based methods directly estimate the exponential delay terms with suitable rational polynomials. After replacing a delay term with a first-order lead-lag block (known as Rekasius's substitution), a finite number of eigenvalues of DCPPS located on the imaginary axis over the whole delay span $[0, \infty)$ can be exactly computed by further employing the Routh stability criterion [11]. In addition, the Padé rational polynomial is popular in approximating exponential delay terms, so that efficient stability analysis [12] and controller design [13–17] can be achieved. It is known that the Padé approximation generally yields good phase approximation, but introduces a non-minimum phase artifact in the initial transient response. For a system with commensurate delays, its eigen-spectrum can be explicitly expressed by using the Lambert–W function if system state matrices can be simultaneously triangularized [18]. Obviously, the Lambert–W function method only suits for a particular class of time delay systems.

Spectral discretization-based methods have been emerged as another way for efficient eigen-analysis of DCPPS. The methods are characterized by discretization of two spectral operators, i.e., solution operator associated with the DCPPS and infinitesimal generator of the solution operator semigroup. Spectral discretization-based methods have been extensively studied in the fields of numerical analysis and computational mathematics during the past decade [10]. Several relevant MATLAB toolboxes are also available, including DDE-BIFTOOL [19], TRACE-DDE [20], etc. Up to the recent few years, the methods have been applied to power engineering community. The Chebyshev discretization of infinitesimal generator-based method (abbreviated as IGD) suggested in [21] was employed by Milano et al. to compute the eigenvalues of DCPPS with single time delay, so that their impacts on system small signal stability were evaluated [22]. In [23], IGD was further compared with linear multi-step and Runge–Kutta discretization scheme of solution operator (abbreviated as SOD-LMS and SOD-RK, respectively) in computing the rightmost eigenvalues of large DCPPS with multiple delays. Numerical studies revealed that IGD was more accurate than SOD-LMS/RK and with less computational burden. It is noteworthy that a GPU-based parallel implementation of the Shur method and QR factorization was employed to speed up the computation. To achieve this, both high-performance computer and sophisticated programming skills were required.

1.2. Motivation and main work

From viewpoints of the authors of this paper, the applications of IGD presented in [22,23] have two major shortcomings, preventing themselves from analyzing large DCPPS with high efficiency. On the one hand, low frequency oscillation modes rather than rightmost eigenvalues are of more interests for power engineers. Preconditioning techniques, such as shift-invert and Cayley transforms [24], which are indispensable to achieve this aim, are actually absent from IGD. In this paper, this is overcome by applying the shift-invert transform to the wanted low frequency oscillation modes so that they become dominant in moduli. On the other hand, IGD does not exploit the sparsities in infinitesimal generator's discretized matrix and in augmented system state matrices [25], resulting in huge computational burden and even prohibitive memory demand.

Different from the parallel implementation adopted in [23], in our opinion, the root measure to solve this problem is to adopt the sparsity-oriented eigenvalue algorithms and compute critical low frequency oscillation of DCPPS. The sparse eigenvalue computation is characterized by the product between the discretized matrix's inversion and vector. However, this is challenged by the fact that the discretized matrix's inversion is implicit about system state matrices. In general, there are two ways to remedy this drawback of IGD and endow the method with scalability for analyzing large DCPPS, depending on solving the involved matrix-inversion and vector product by either direct or iterative method.

The direct solver for the matrix-inversion and vector product in IGD is presented in our previous work [26] by designing a new mesh grid to discretize the infinitesimal generator of DCPPS, leading to a highly structured approximate matrix. This allows one to explicitly represent the inverse of the discretized matrix with highly sparse augmented state matrices of DCPPS. The subsequent matrix-inversion and vector product therefore can be directly and efficiently implemented.

Alternatively, in this paper, the mesh grid and the resultant discretized matrix of the infinitesimal generator remain unchanged. The original IGD is further developed to endow itself with scalability for analyzing large DCPPS by iteratively computing the matrix-inversion and vector product involved in the sparse eigenvalue computation. The presented iterative IGD (i.e., IIGD) consists of three core techniques. First, the sparsity of infinitesimal generator's discretized matrix is exploited by factorizing its dominant block into sum of Kronecker products between constant Lagrange vectors and system state matrices. Second, the shift-invert preconditioning technique is applied to transform the required eigenvalues into those dominated in moduli. Third, a novel subspace-based IDR (induced dimension reduction) method is adopted to efficiently and iteratively calculate the matrix-inversion and vector product, where the unique property of the Kronecker product and inherent sparsities in augmented system state matrices are utilized to guarantee efficiency and scalability.

1.3. Contribution

The presented IIGD offers general significance for spectral discretization-based eigen-analysis of large DCPPS.

On the one hand, the rationale behind the explicit reformulation of the infinitesimal generator's discretized matrix suits for the discretized matrices of all spectral operators of DCPPS under various discretization schemes. It lays the basis of exploiting the sparsities in both the discretized matrix and augmented system state matrices.

On the other hand, the efficient and iterative solver for the product between inverse of the infinitesimal generator's discretized matrix and vector avoids the infeasibility in analyzing large DCPPS. Essentially, the basic idea behind the iterative solution suits for spectral operators of DCPPS under most discretization schemes, where the inverses of the resultant discretized matrices are only with ordinary logic structures and implicit about system state matrices, e.g., solution operator discretization with Runge–Kutta and Chebyshev schemes [27,28], respectively.

1.4. Outline

The remainder of the paper is as follows. Section 2 models the DCPPS and formulates its eigen-problem. Section 3 gives the theoretical background of the IGD method. Section 4 elaborates the IIGD method. The effectiveness of IIGD is validated in Sections 5 and 6, followed by Section 7 that draws the conclusion of the paper.

2. DCPPS modeling and eigen-problem formulation

This section establishes the model of DCPPS and formulates its eigenvalue computation problem.

2.1. Physical nature of DCPPS

Fig. 1 illustrates the physical power system and the associated wide-area measurement and damping control system. It is clear that the closed-loop power system is a cyber-physical system [1].