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AC OPF in radial distribution networks – Part I: On the limits of the branch flow convexification and the alternating direction method of multipliers

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ABSTRACT

The optimal power-flow problem (OPF) has always played a key role in the planning and operation of power systems. Due to the non-linear nature of the AC power-flow equations, the OPF problem is known to be non-convex, therefore hard to solve. During the last few years several methods for solving the OPF have been proposed. The majority of them rely on approximations, often applied to the network model, aiming at making OPF convex and yielding inexact solutions. Others, kept the non-convex nature of the OPF with consequent increase of the computational complexity, inadequateness for real time control applications and sub-optimality of the identified solution. Recently, Farivar and Low proposed a method that is claimed to be exact for the case of radial distribution systems under specific assumptions, despite no apparent approximations. In our work, we show that it is, in fact, not exact. On one hand, there is a misinterpretation of the physical network model related to the ampacity constraint of the lines' current flows. On the other hand, the proof of the exactness of the proposed relaxation requires unrealistic assumptions and, in particular, (i) full controllability of loads and generation in the network and (ii) no upper-bound on the controllable loads. We also show that the extension of this approach to account for exact line models might provide physically infeasible solutions. In addition to the aforementioned convexification method, recently several contributions have proposed OPF algorithms that rely on the use of the alternating direction method of multipliers (ADMM). However, as we show in this work, there are cases for which the ADMM-based solution of the non-relaxed OPF problem fails to converge. To overcome the aforementioned limitations, we propose a specific algorithm for the solution of a non-approximated, non-convex OPF problem in radial distribution systems. In view of the complexity of the contribution, this work is divided in two parts. In this first part, we specifically discuss the limitations of both BFM and ADMM to solve the OPF problem.

1. Introduction

The category of optimal power-flow problems (OPFs) represents the main set of problems for the optimal operation of power systems. The first formulation of an OPF problem appeared in the early 1960s and has been well-defined ever since [1]. It consists in determining the operating point of controllable resources in an electric network in order to satisfy a specific network objective subject to a wide range of constraints. Typical controllable resources considered in the literature are generators, storage systems, on-load tap changers (OLTC), flexible AC transmission systems (FACTS) and loads (e.g., [2–6]). The network objective is usually the minimization of losses or generation costs, and typical constraints include power-flow equations, capability curves of the controllable resources, as well as operational limits on the line power-flows and node voltages (e.g., [7]).

The OPF problem is known to be non-convex, thus difficult to solve efficiently (e.g., [8–10]). Since the problem was first formulated, several techniques have been used for its solution.

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Among others, non-linear and quadratic programming techniques, Newton-based methods, interior point methods in the earlier years, as well as heuristic approaches based on genetic algorithms, evolutionary programming, and particle-swarm optimization in recent years (e.g., [11–14]). These techniques, even though they have been shown to successfully solve instances of the non-convex OPF problem, seek to find a local optimal solution of the OPF. They, generally, utilize powerful general purpose solvers or in-house developed software but they cannot guarantee the identification of the global optimal solution. In general, they are characterized by high computational complexity. The first category of approaches makes use of gradient-based optimization algorithms or even requires the use of Hessian matrices related to the problem. Therefore, such techniques require several assumptions on the OPF problem formulation such as analytic and smooth objective functions. Heuristics have been applied widely in the literature as a solution technique, for instance in cases where the OPF problem is non-smooth, non-differentiable and highly non-linear.

Recently, the OPF problem is becoming more compelling due to the increasing penetration of embedded generation in distribution networks, essentially composed by renewable resources.[1] The distributed nature of such resources, as well as their large number and potential stochasticity increase significantly the complexity and the size of the OPF problem and bring about the need for distributed solutions. In this direction, several algorithms have been proposed in the literature to handle large-scale OPF problems (e.g., [15–17]). Additionally, several contributions have proposed specific distributed algorithms for the solution of the OPF problem. In [18,19] the authors design a dual-ascent algorithm for optimal reactive power-flow with power and voltage constraints. In [20,21] dual decomposition is used as the basis for the distributed solution of the OPF problem. Finally, a significant number of contributions propose distributed formulations of the OPF problem that are based on the alternating direction method of multipliers (ADMM) (e.g., [22,20,23–26]).

Recently a lot of emphasis is put on the convexification of the OPF problem. The reason behind this emerging trend is that convex problems provide convergence guarantees to an optimal solution and therefore such methods can be deployed within the context of control applications for power systems and specifically distribution networks. However, most of the proposed convexification schemes either do not guarantee to yield an optimal solution or they are based on approximations that convexify the problem in order to guarantee convergence. These approximations, often, either lead to (i) misinterpretation of the system model [27] or (ii) solutions that, even though mathematically sound, might be far away from the real optimal solution, thus having little meaning for the grid operation [28].

Recently, Farivar and Low proposed in [29,30] a convexification of the problem that is claimed to be exact for radial networks. In Part I of this paper, we show that this claim is not exact, as the convexification of the problem leads to an inexact system model. We also show that the method of ADMM-based decomposition, which comes together with the convexification, does not work for a correct system model. In this first part of the paper we focus on the Farivar-Low convexification and ADMM algorithms since they are considered as the most prominent ones by the recent literature on the subject. As an alternative, we propose in Part II an algorithm for the solution of the correct AC OPF problem in radial networks. Like ADMM, it uses an augmented Lagrangian, but unlike ADMM, it uses primal decomposition [31] and does not require that the problem be convex. We consider a direct-sequence representation of the electric distribution grid and we present both a centralized and a decentralized asynchronous version of the algorithm.

The structure of this first part is as follows. In Section 2 we present the generic formulation of the OPF problem in radial distribution systems and we classify several OPF algorithms based on the approximations and assumptions on which they rely. In Section 3 we discuss the limitations and applicability of the Farivar-Low formulation of the OPF problem proposed in [29,30]. We provide, in Section 4, the ADMM-based solution of the original non-approximated OPF problem. In the same section, we highlight specific cases where the ADMM-based algorithm fails to converge. Finally, we provide the main observations and concluding remarks for this part in Section 5.

2. Generic formulation of the OPF problem

2.1. Notation and network representation

In the rest of the paper, we consider a balanced radial network composed of buses ($\mathcal{B}$), lines ($\mathcal{L}$), generators ($\mathcal{G}$) and loads ($\mathcal{C}$). The network admittance matrix is denoted by Y. Several generators/loads can be connected to a bus $b \in \mathcal{B}$. We denote that a generator $g \in \mathcal{G}$ or a load $c \in \mathcal{C}$ is connected to a bus by "$g \in b$" and "$c \in b$". We assume that the nodal-power injections are voltage-independent. A line $\ell \in \mathcal{L}$ is represented using its exact π-equivalent model and it has a receiving and a sending end denoted by ℓ^+ and ℓ^-. Each line is connected to two adjacent buses: $\beta(\ell^+)$ and $\beta(\ell^-)$, respectively. $\bar{Y}_\ell$ denotes the longitudinal admittance of a line, $\bar{Y}_{\ell_0^+}$ ($\bar{Y}_{\ell_0^-}$) is the shunt capacitance at the receiving (sending) end of the line.[2] The notation adopted is shown in detail in Fig. 1 where the network branch connecting the generic network nodes i and j is represented.

2.2. Generic OPF formulation

The traditional formulation of the OPF problem consists in minimizing a specific network objective:

$$\min_{\bar{S}_g,\bar{S}_c,\bar{S}_\ell^+,\bar{S}_\ell^-,\bar{I}_\ell^+,\bar{I}_\ell^-,\bar{V}_b} \sum_{g\in\mathcal{G}} C_g(\bar{S}_g) + \sum_{c\in\mathcal{C}} C_c(\bar{S}_c) \quad (1)$$

The first term of the network objective (C_g) in (1) is typically a non-decreasing convex function accounting for the minimization of the generation costs or the network real power losses. The second term (C_c) is included in the objective when the cost of non-supplied load is taken into account.

The following set of constraints is considered[3]:

$$\sum_{g\in b}\bar{S}_g - \sum_{c\in b}\bar{S}_c + \sum_{\beta(\ell^+)=b}\bar{S}_{\ell^+} + \sum_{\beta(\ell^-)=b}\bar{S}_{\ell^-} = 0, \quad \forall b \in \mathcal{B} \quad (2)$$

$$\bar{S}_{\ell^+} = \bar{V}_{\beta(\ell^+)}\underline{I}_{\ell^+}, \quad \bar{S}_{\ell^-} = \bar{V}_{\beta(\ell^-)}\underline{I}_{\ell^-}, \quad \forall \ell \in \mathcal{L} \quad (3)$$

$$\bar{I}_{\ell^+} = \bar{Y}_\ell(\bar{V}_{\beta(\ell^+)} - \bar{V}_{\beta(\ell^-)}) + \bar{Y}_{\ell_0^+}\bar{V}_{\beta(\ell^+)}, \quad \forall \ell \in \mathcal{L} \quad (4)$$

[1] It is worth noting that transmission and distribution systems are different with respect to (i) topology, (ii) electrical line parameters, (iii) power flow values, (iv) nature and number of controllable devices. Therefore, these systems require dedicated OPF algorithms that account for their specific characteristics. The focus of this work is on OPF algorithms specifically designed for the case of distribution networks.

[2] In the rest of the paper, complex numbers are denoted with a bar above (e.g., $\bar{V}$) and complex conjugates with a bar below (e.g.,$\underline{V}$).

[3] Note that the proposed formulation can be extended without loss of generality to the case of multi-phase unbalanced grids by adopting the so-called compound network admittance matrix, (i.e., the 3-phase representation of the grid model which takes into account the various couplings between the network phases) instead of the single-phase equivalents. In this case, each of the constraints in (2)–(9) needs to be formulated separately for each network phase.