Electric Power Systems Research xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Data fusion-based distributed Prony analysis

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ARTICLE INFO

Article history:
Received 5 August 2016
Received in revised form 12 October 2016
Accepted 21 October 2016
Available online xxx

Keywords:
Prony analysis
Data fusion
Kalman filter
Distributed computing

ABSTRACT

This paper presents a distributed Prony analysis algorithm using data fusion approach. This classic approach can be found in Kalman filter's measurement update. Distributed optimization algorithms, e.g., alternating direction method of multipliers (ADMM), suitable for constrained optimization problems, have been proposed in the previous literature to develop distributed architecture. In this article, we show that Prony analysis, a non-constrained least square estimation (LSE) problem, can be solved using the classic data fusion approach. Compared to the iterative distributed optimization algorithms (e.g., ADMM and subgradient methods), data fusion takes only one step. There is no need for iteration and there is no issue related to convergence. This approach leads to a distributed Prony analysis architecture which requires a much less demanding communication system (the bandwidth can be less than 0.1 Hz) compared to the conventional centralized Prony analysis for multiple channels which requires a bandwidth of 30 Hz. The application discussed in this paper is to identify oscillation modes from real-world phasor measurement unit (PMU) data and further reconstruct signals. A key technical challenge to implement Prony analysis for signals from multiple channels is the difficulty to identify the noise characteristics of each channel. In this paper, a method is proposed to identify the noise covariances, which leads to the construction of a weighted least square estimation (WLSE) problem. This problem is solved through a distributed architecture. The effectiveness of the proposed distributed Prony analysis is demonstrated through case study results. The accuracy of the estimation is improved in one order compared with the centralized Prony analysis.

1. Introduction

A power system is a massive system that can be perturbed by load changes, generator trips, faults or networks changes. Power system oscillations are common issues. To mitigate oscillations, oscillations should be identified and studied in a timely manner. There are two separate approaches to identify power system oscillations. The first approach is based on detailed dynamic models of the system. State-space modeling and the eigenvalue analysis can give the system's oscillation modes [1]. Detailed modeling of a complicated power system is challenging and prone to errors. The second approach is based on measurements to identify oscillation modes. Measurement-based approach has been adopted by control engineers in practice. For example, equivalent system models will be constructed based on the measurements and further control strategies will be developed based on the identified system models.

With phasor measurement unit (PMU) data collected, electromechanical oscillation modes (<2 Hz) can be identified from these measurements at 30 Hz sampling rate. Several measurement-based system identification have been proposed for PMU data-based estimation, such as Kalman filters [2–4], least square estimation [5], and subspace algorithm [6]. Prony analysis is one of the most common measurement-based identification approaches to identify oscillatory modes. Prony analysis has been introduced by Hauer et al. in power systems in 1990 [7,8]. The main idea is to directly estimate the frequency, damping and phase of modal components of a measured signal. An extension to Prony analysis is then introduced which allowed multiple signals to be analyzed at the same time resulting [9].

Application of distributed optimization techniques has recently been introduced in system modes identification [10–12]. For example, in [10], distributed Prony analysis using alternating direction method of multipliers (ADMM) has been combined with centralized Prony method to estimate the slow frequency eigenvalues. Simulation data generated by PST [13] toolbox of IEEE 39-bus system is used to conduct Prony analysis. In the author's previous paper [14], another distributed Prony analysis algorithm using consensus and subgradient update is developed. Distributed Prony analysis presented in the aforementioned papers can be applied to multiple signals from multiple locations collected at the same

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http://dx.doi.org/10.1016/j.epsr.2016.10.052
0378-7796/

period of time. These algorithms can handle a large-dimension of PMU data by solving least square estimation (LSE) problems with small sizes in parallel and iteratively. This paper serves as a rebuttal of the above distributed optimization approaches: iterations are not necessary. Indeed, Prony analysis is essentially an LSE problem without any constraints. Prony analysis of multi-channel signals is a multi-objective LSE problem. LSE problems were introduced by Gauss in 1790s. In 1960s, R. Kalman designed an iterative approach for LSE. See [15] for a detailed description. In Kalman filter, measurement update takes one step to find the best estimate given the prior information and current measurement [16]. There is no need of iteration. Compared to Kalman filter-based approach, distributed optimization approaches are not efficient. Kalman filter-based approach has been used in multi-sensor data fusion [17]. In this paper, the approach is named as data fusion approach.

In this paper, the philosophy of data fusion is examined in detail and applied to develop an effective algorithm for distributed Prony analysis. A key technical challenge to implement Prony analysis for signals from multiple channels is the difficulty to identify the noise characteristics of each channel. In this paper, a method is proposed to identify the noise covariances, which leads to the construction of a weighted least square estimation (WLSE) problem. This problem is solved through a distributed architecture. In a nutshell, the contribution of the paper is to implement Kalman filter-based data fusion approach in Prony analysis with multiple channels. This approach has not been seen in Prony analysis. Compared to the other approaches where constrained optimization problems are formulated and further been solved by iterative distributed algorithms, e.g., [14,12], the proposed approach does not require iterations and has advantages in computation.

The rest of the paper is as follows. Section 2 describes the fundamentals of Prony analysis. Section 3 describes the centralized multi-channel Prony analysis. Section 4 presents data fusion and distributed Prony analysis. Section 5 further examines the relationship of data fusion based Prony analysis and multi-channel Prony analysis. Section 6 presents the case study results. Section 7 concludes this paper.

2. Fundamentals of Prony analysis

Consider a Linear-Time Invariant (LTI) system with the initial state of $x(t_0)=x_0$ at the time t_0, if the input is removed from the system, the dynamic system model can be represented as [18]:

$$\dot{x}(t) = Ax(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $y \in \mathbb{R}$ is defined as the output of the system, $x \in \mathbb{R}^n$ is the state of the system, $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{1\times n}$ are system matrices. The order of the system is defined by n. If the λ_i, p_i, and q_i are the ith eigenvalue, the corresponding right eigenvector, and left eigenvectors of A respectively, (1) can be expressed as:

$$x(t) = \sum_{i=1}^{n}(q_i^T x_0)p_i e^{\lambda_i t} = \sum_{i=1}^{n} R_i x_0 e^{\lambda_i t} \tag{3}$$

where x_0 is the initial state and $R_i = p_i q_i^T$ is a residue matrix. Based on (2), the $y(t)$ can be expressed as:

$$y(t) = \sum_{i=1}^{n} CR_i x_0 e^{\lambda_i t}. \tag{4}$$

The observed or measured $y(t)$ consists of N samples which are equally spaced by Δt as: $y(t_k)=y(k)$, $k=1, \ldots, N-1$. The basic assumption is to consider the signal record to be noise free and the order of the system can be set as: $n=\text{floor}(N/2)$ [7]. Therefore, (4) can be recasted in the exponential form as:

$$\hat{y}(t_k) = \sum_{i=1}^{n} B_i e^{\lambda_i k\Delta t} \tag{5}$$

$$= \sum_{i=1}^{n} B_i z_i^k, \qquad k = 1, \ldots, N \tag{6}$$

where $B_i=CR_i$, N is the number of samples, z_i are the eigenvalues of the system in discrete time domain, and B_i is the residue of z_i. z_i can be expressed as:

$$z_i = e^{\lambda_i \Delta t} \tag{7}$$

Due to the fact that $k=1, \ldots, N$, (6) can be expressed in matrix form as:

$$\begin{bmatrix} B_1 z_1^0 + \cdots + B_n z_n^0 \\ B_1 z_1^1 + \cdots + B_n z_n^1 \\ \vdots \\ B_1 z_1^{N-1} + \cdots + B_n z_n^{N-1} \end{bmatrix} = \begin{bmatrix} \hat{y}(0) \\ \hat{y}(1) \\ \vdots \\ \hat{y}(N-1) \end{bmatrix}. \tag{8}$$

Or in a simple form: $ZB=Y$ as shown in (9).

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_n^0 \\ z_1^1 & z_2^1 & \cdots & z_n^1 \\ \vdots & \vdots & \vdots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} \hat{y}(0) \\ \hat{y}(1) \\ \vdots \\ \hat{y}(N-1) \end{bmatrix} \tag{9}$$

As the z_i are the roots of the characteristic polynomial function of the system, in order to find the z_i, the coefficients of the polynomial need to be found first. The polynomial is formed as:

$$z^n - (a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n z^0) = 0. \tag{10}$$

While the roots z_i might be complex numbers, the system polynomial coefficients a_i are real numbers. This feature helps develop algorithms since real numbers will be handled by computer algorithms while complex numbers cannot be directly handled.

From (10), we have

$$z^n = a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n z^0. \tag{11}$$

Further, a linear prediction model (12) can be formulated since $y(k)$ is the linear combination of $z_i(k)$ based on (6). Therefore,

$$y(n) = a_1 y(n-1) + a_2 y(n-2) + \cdots + a_n y(0). \tag{12}$$

Enumerating the signal samples from step n to step N, we have (13): $Y=Da$.

$$\underbrace{\begin{bmatrix} y(n) \\ \vdots \\ y(n+k) \\ \vdots \\ y(N) \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} y(n-1) & \cdots & y(0) \\ \vdots & \ddots & \vdots \\ y(n+k-1) & \cdots & y(k) \\ \vdots & \ddots & \vdots \\ y(N-1) & \cdots & y(N-n) \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} a_1 \\ \vdots \\ a_k \\ \vdots \\ a_n \end{bmatrix}}_{a} \tag{13}$$

The best estimate of a is found from the following normal equation.

$$\hat{a} = (D^T D)^{-1} D^T Y. \tag{14}$$

In the computing implementation, direct matrix inversion may result in numerical inaccuracy when the matrix $D^T D$ approaches