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Recursive and non-recursive algorithms for power system real time phasor estimations

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ABSTRACT

This work presents some of the computational algorithms used for phasor estimations in Electrical Power Systems. The IEEE C37.118.1 standard establishes the phasor estimation structure and performance, but does not define the algorithm itself to be used. Considering this, various methods can be adopted provided that the standard precision is met. Some estimation algorithms presented in the literature were compared in this paper, and their behavior was evaluated for some test cases. The methods: Discrete Fourier Transform, Recursive Discrete Fourier Transform, Least Squares, Recursive Least Squares, Discrete Wavelet Transform and Recursive Wavelet Transform were tested using synthetic signals, evaluating the Total Vector Error, response and delay times, as well as overshoot. The algorithms were also embedded in hardware and evaluated by simulated signals, using the outputs of a Real Time Digital Simulator.

1. Introduction

Phasors are basic tools to analyze Alternating Current (AC) circuits and they can be used in communication, control and power systems. In power systems, the voltage and current phasors are obtained following a mathematical procedure, using the signal samples measured by specific equipment, known as Phasor Measurement Units (PMUs). These measures can provide the power flow, stability analysis, state estimation, protection parameter definition, etc.

Phasors measured at the same instant in time are called synchrophasors, and the system's structure and procedures required for their estimation are defined in the IEEE C37.118.1 standard [1]. This standard defines the time tag, synchrophasor messaging patterns, as well as tests and parameters to evaluate the measuring process.

Despite the fact that the IEEE C37.118.1 standard establishes the requirements for the measuring process, it does not define which specific method should be used for phasor estimations. Therefore, various methods have been proposed in the literature, and this paper will address some of the main techniques available for phasor estimation.

The Discrete Fourier Transform (DFT) and the Least Squares (LSQ) methods were first used to estimate phasors in the early digital relays. Ref. [2] was one of the first studies to address those methods for PMU estimations, which proposed the use of DFT and Recursive DFT (RDFT) for phasor estimations. The DFT and LSQ are still widely used for phasor estimation in power systems [3].

The Discrete Wavelet Transform (DWT) was also used for the same purpose in its recursive [4,5] and non-recursive [6] form. Kalman Filters [7] and Phase-Locked Loop (PLL) [8] were used to estimate phasors and frequency simultaneously. Fuzzy Logic [9], Artificial Neural Networks [10,11] and Genetic Algorithms [12] were also used as phasor estimators, obtaining accurate responses at the expense of their complexity.

Some recent research addresses improved phasor estimation for specific applications, such as dynamic voltage restoration [13], fault location in series-compensated lines [14], accurate phasor estimation during power swings [15,16], time-varying frequency events [17,18], as well as system protection [19].

Despite the wide range of methods proposed in the literature, the advantages of new techniques should be investigated and compared to traditional ones. In [20], for example, the authors proposed an analysis comparing DWT and DFT for phasor estimation in a digital relay. They concluded that the computational power of the DWT is similar to the DFT (implemented using the Fast Fourier Transform), with no advantage to the DWT.

Within this context, this study aims to compare some methods for phasor estimations found in the literature. The following

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methods were used: DFT and RDFT [2], LSQ [21], Recursive Least Squares (RLS) [22], DWT [6] and Recursive Wavelet Transform (RWT) [4]. They were evaluated under permanent and transient conditions, using the tests defined in [1]. The algorithms were also embedded in hardware to analyze phasor estimation during some events simulated via a Real Time Digital Simulator (RTDS). The processing time of the studied methods was also analyzed.

2. Studied methods

The basic theory of the methods compared in this paper will be presented as follows. More details about each technique can be found in Refs. [2,4,6,21,22].

The fundamental frequency of power system signals are expressed as a sinusoid function, with amplitude $\sqrt{2}X$, phase ϕ, and frequency f_0.

$$x(t) = \sqrt{2}Xsin(2\pi f_0 t + \phi) \tag{1}$$

The phasor of signal (1) can be expressed as

$$\bar{X} = X\angle\phi = Xcos\phi + jXsin\phi \tag{2}$$

2.1. Discrete Fourier Transform [2]

If signal (1) is sampled by using a rate of N samples per cycle, each sample k is expressed as

$$x_k = \sqrt{2}Xsin\left(\frac{2\pi}{N}k + \phi\right) \tag{3}$$

Given N samples, relative to one cycle of the signal, the fundamental frequency phasor (2) can be calculated by using the DFT.

$$\bar{X} = j\frac{\sqrt{2}}{N}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}k} \tag{4}$$

2.2. Recursive Discrete Fourier Transform [2]

The fundamental frequency phasor of a certain data window is calculated by (4). The phasor of the subsequent window is calculated by the same expression, by changing the sum limits to $1 \to N$, which contains $N-1$ samples similar to the previous window ($0 \to N-1$). Due to this feature, the phasor of the previous window can be used to estimate the next one, by excluding one sample (previous window) and adding the new one. If $\bar{X}_{r-1}$ is the previous phasor, x_{N+r} is the new sample obtained and x_r is the old one, the new phasor $\bar{X}_r$ is estimated by

$$\bar{X}_r = \bar{X}_{r-1} + j\frac{2}{\sqrt{2}N}(x_{N+r} - x_r)e^{-j\frac{2\pi}{N}(r-1)} \tag{5}$$

2.3. Least Squares [21]

The fundamental frequency phasor is determined by using vector V, which is calculated by (6), where S is the vector containing the data window of one cycle of the signal and A^p the pseudoinverse of the matrix A. Elements V_1 and V_2 of V contain the exponential decaying component, while elements V_3 and V_4 represent the real and imaginary parts of the phasor, such as $\bar{X} = V_3 + jV_4$.

$$V = A^p \times S \tag{6}$$

A has dimension $(N;2M+1)$, and V $(2M+1;1)$, where M is the highest harmonic considered, which is determined by the cut-off frequency of the anti-aliasing filter. The A elements are given in (7), where m is a harmonic, defined between $2 \to M$, and n is the window sample ($1 \to N$).

$$\begin{aligned} &a_{n,1} = 1; \quad a_{n,3} = \sin\left(\frac{2\pi}{N}k\right); \quad a_{n,2m+1} = \sin\left(m\frac{2\pi}{N}k\right); \\ &a_{n,2} = \frac{k}{Nf_0}; \quad a_{n,4} = \cos\left(\frac{2\pi}{N}k\right); \quad a_{n,2m+2} = \cos\left(m\frac{2\pi}{N}k\right). \end{aligned} \tag{7}$$

2.4. Recursive Least Squares [22]

The magnitude X and phase ϕ of the phasor are given by vector B (8), recursively calculated by using (9). A vector of zeros can be adopted as the initial B.

$$B(k) = [X(k) \;\; \phi(k)] \tag{8}$$

$$B(k) = B(k-1) - e(k)R(k)U(k) \tag{9}$$

$e(k)$ is the error between the measured sample ($x(k)$) and the estimated ($\hat{x}(k)$), expressed as

$$e(k) = \hat{x}(k) - x(k) \tag{10}$$

U is the coefficient vector, expressed as

$$U(k) = \left[\sin\left(\frac{2\pi}{N}k + \hat{\phi}(k)\right) \; \hat{X}(k)\cos\left(\frac{2\pi}{N}k + \hat{\phi}(k)\right)\right] \tag{11}$$

R replaces the pseudoinverse of U and is recursively updated by using (12). According to [22], the use of an identity matrix is recommended for the initial R, and $0 \ll \lambda < 1$.

$$R(k) = \frac{1}{\lambda}\left[R(k-1) - \frac{R(k-1)U(k)U(k)^T R(k-1)}{\lambda + U(k)^T R(k-1)U(k)}\right] \tag{12}$$

2.5. Discrete Wavelet Transform [6]

The DWT does not provide magnitude and phase directly, requiring the wavelet decomposition of the measured signal (x) and reference signals (r_1 and r_2). The first reference signal is a sine function, defined as (13), while the second is a cosine, given by (14). These signals must have N samples, such as the measured signal data window.

$$r_1(k) = \sin(2\pi k/N) \tag{13}$$

$$r_2(k) = \cos(2\pi k/N) \tag{14}$$

Signals (1), (13) and (14) are filtered by Wavelet, using the Haar function (other functions may be adopted) in order to obtain the approximation at the second level of x, r_1 and r_2: Ax^2, Ar_1^2 and Ar_2^2; and their Euclidean norm: $|Ax^2|$, $|Ar_1^2|$ and $|Ar_2^2|$, respectively. These are used to calculate angles θ_1 and θ_2, according to (15) and (16).

$$\theta_1 = \cos^{-1}\left(\frac{Ar_1^2 \cdot Ax^2}{|Ar_1^2||Ax^2|}\right) \tag{15}$$

$$\theta_2 = \cos^{-1}\left(\frac{Ar_2^2 \cdot Ax^2}{|Ar_2^2||Ax^2|}\right) \tag{16}$$

Phase ϕ is determined by

$$\begin{cases} \phi = \theta_1, & \text{if}\,\theta_2 \le \pi/2 \\ \phi = 2\pi - \theta_1, & \text{if}\,\theta_2 > \pi/2 \end{cases} \tag{17}$$

Another reference signal (r_3) with phase ϕ is defined by

$$r_3(k) = \sin(2\pi k/N + \phi) \tag{18}$$