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Electromagnetic fields of accelerating charges: Applications in lightning protection

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ABSTRACT

Electromagnetic fields generated by accelerating charges can be utilized to evaluate the electromagnetic fields generated by systems where moving charges and/or propagating currents are present. The technique can be used easily to evaluate the electromagnetic fields generated by systems in which propagating currents are present. This is illustrated by utilizing the equations to derive expressions for the electromagnetic fields generated by systems in which current pulses injected by lightning flashes are propagating.

1. Introduction

In the literature, three procedures have been used to calculate the electromagnetic fields from return strokes [1,2]. In the first technique of these, the source is described only in terms of current density and the fields are expressed entirely in terms of the return stroke current. In the second technique, the source is expressed in terms of the current and the charge densities and the fields are given in terms of both the current and the charge density. In the third technique, the fields are expressed in terms of the apparent charge density. The fields are connected to the source terms through the vector and scalar potentials. A fourth technique was introduced recently by Cooray and Cooray [3]. In that technique the standard equations for the electromagnetic fields generated by an accelerating charge are utilized to evaluate the electromagnetic fields from lightning return strokes. They showed how this technique can be applied to calculate the electromagnetic fields predicted by different return stroke models. Cooray and Cooray [4] applied these equations to re-derive the electromagnetic fields from short dipoles. They have also applied these equations to give a physical interpretation to the TEM fields of horizontal conductors located over ground [5]. The same procedure was applied by the same authors to calculate the electromagnetic fields produced by electron avalanches [6,7]. This procedure has several advantages over other conventional methods of electromagnetic field calculations when the system under consideration can be represented by a set of transmission lines in air. In this case the complete electromagnetic field can be reduced to a sum of electromagnetic radiation fields generated at the end points of the transmission lines. This will reduce the computational time and at the same time gives a clear physical picture of the events that led to the various parts of the electromagnetic fields.

In the applications related to lightning protection we are mostly interested in evaluating the electromagnetic fields generated by a current pulse moving from one point in space to another with the speed of light. Expressions for these field components have been derived and published already by Cooray and Cooray [3,4]. The goal of this paper is to show how these field expressions could be utilized in several applications pertinent to lightning research.

The paper is organized as follows. In the first part of the paper a comparison is made between the results obtained using traditional way of calculating the electromagnetic fields generated by propagating current pulses with the results obtained using charge acceleration equations. After illustrating that both procedures will generate identical electromagnetic fields the use of this technique in several problems pertinent to lightning protection studies are presented.

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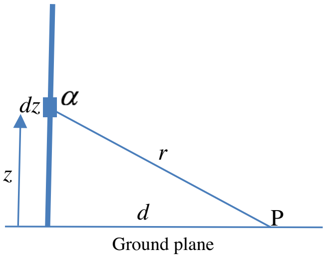

Fig. 1. Geometry relevant to the calculation of electric field generated by a lightning channel located over a perfectly conducting ground plane using the dipole approximation.

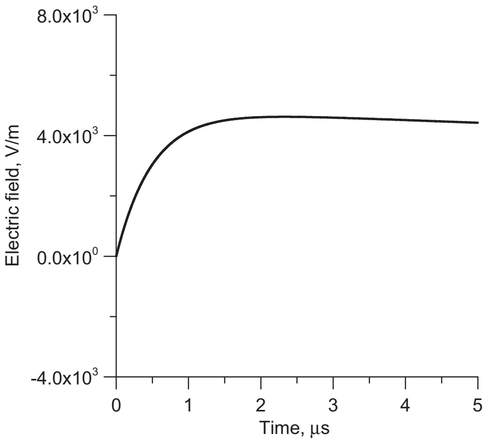

Fig. 2. The electric field (directed into the ground) at ground level at a distance of 100 m from the strike as predicted by Eq. (1). The current at the channel base is assumed to be given by $I(t) = I_0\,\{e^{-t/\tau_1} - e^{-t/\tau_2}\}$ with I_0 = 8.18 kA, τ_1 = 50 µs, τ_2 = 0.5 µs.

2. Comparison of dipole technique with the charge acceleration technique using transmission line return stroke model

In the transmission line return stroke model it is assumed that the return stroke can be represented by a current pulse that is initiated at ground level and propagates along the channel with constant speed without attenuation [8]. Let us consider a transmission line model where the speed of propagation of the current pulse is equal to the speed of light. If one use the technique of charge acceleration then the velocity fields go to zero (because$u = c$) and one will end up with only a radiation field. The total vertical electric field at point P located at a horizontal distance d at ground level (which is pure radiation) is given by (see Fig. 1) [3, see also the Appendix A of this paper]

$$E(t) = \frac{i(0, t - d/c)}{2\pi\varepsilon_o cd} \tag{1}$$

Note that this electric field is directed into the ground. One has to introduce a negative sign if the component of the field in the positive z-direction is considered. In lightning research the standard technique to calculate the electric and magnetic fields is to use the dipole approximation. Let us now write down the expressions for the electric field at ground level as given by the dipole approximation. In this approximation the channel is divided into large number of elementary channel sections. Each channel section is treated as a short dipole of length dz and the total electric field is obtained by summing the contribution to the electric field at the point of observation from each dipole. The geometry relevant to the calculation is given in Fig. 1. The corresponding field expressions are the following [1].

$$E(t) = \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{2 - 3\,\sin^2\alpha}{r^3}\int_{t_b}^{t} i(z, \tau - r/c)d\tau dz + \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{2 - 3\,\sin^2\alpha}{cr^2} i(z, \tau - r/c)dz + \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{\sin^2\,\alpha}{c^2 r}\frac{\partial i(z, \tau - r/c)}{\partial t}dz \tag{2}$$

Note that the total field can be written as a sum of three field components, namely, electrostatic (term varying as the inverse cube of the distance), induction (term varying as the inverse square of the distance) and radiation (term varying as the inverse of the distance). In the above equations t_b is the time of arrival of the return stroke front at height z as seen by the observer located at distance d and H is the return stroke height. Even though the mathematical expressions in Eqs. (1) and (2) are very different both give rise to the same electric field. For example the electric field at distance d as predicted by Eq. (1) is shown in Fig. 2 and the one predicted by Eq. (2) but separated into the three components are shown in Fig. 3. Note that both Eqs. (1) and (2) predict the same total electric field. Another interesting point is that in Eq. (1) the total field appears as a radiation term whereas in Eq. (2) it is created by the sum of static, induction and radiation components.

3. Applications in lightning protection

In various applications of lightning protection where electromagnetic field calculations are necessary the basic problem is the calculation of electromagnetic fields generated by a current pulse moving from one point in space to another. In the following paragraphs we will