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A highly stable and accurate computational method for eigensolutions in structural dynamics

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Abstract

A new computational method for the linear eigensolution of structural dynamics is proposed. The eigenvalue problem is theoretically transformed into a specific initial value problem of an ordinary differential equation. Based on the physical meaning of the sign count of the dynamic stiffness matrix, a stability control device is designed and combined with the fourth-order Runge–Kutta method. The resulting method finds the eigenvalues and eigenvectors at the same time, with high accuracy and high stability. Numerical examples show that the proposed method still gives high accuracy solutions when there is a great difference in magnitude among the eigenvalues, and also when some eigenvalues are very close to each other.
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1. Introduction

Since the natural frequencies and modes of vibration take an essential role in structural dynamics, it is a very important aim to have reliable and accurate methods for solving the eigenvalue problem of a structure. However, the computation of structural eigenvalues and eigenvectors remains a difficult task even now. Difficulties arise from both the discretization of the structure (e.g., by the finite element method) and the procedures of numerical analysis. When the higher natural frequencies of a structure are desired, smaller elements should be used [1–4], resulting not only in an increased order of the matrices representing the structure but also in an increased propensity of these matrices for ill-conditioning. Therefore, highly stable and high accuracy computational methods for eigensolutions of structures are necessary to obtain the eigenpairs with good precision.

Among existing numerical methods for the eigensolution, the subspace iteration method has been widely used [5–11]. In spite of its many capabilities, sometimes this method is sensitive to the initial trial values of eigenvalues and eigenvectors and some eigenpairs of interest may be missed as a result of numerical instability.

As an alternative to finding the eigenvalues directly, a few methods [12,13], such as the Wittrick–Williams algorithm [13–17], try to find bounds on the eigenvalues by counting the number of eigenvalues exceeded at each of a sequence of trial frequencies. The procedure to locate the bounds employs only the signs of the diagonal elements of the upper triangular matrix resulting from Gauss elimination of the dynamic stiffness matrix. As a result, the method is so stable that it has been regarded as infallible [13]. However, if the sequence of trial frequencies is chosen by the bisection method, only linear

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convergence on the eigenvalues can be achieved, and the solution of eigenvectors does not benefit from the method. Although efforts to improve the convergence and the computation of eigenvectors have been made by some authors [16–18], further work is needed in this area.

Such root-counting methods have generally been applied to the transcendental eigenvalue problems arising from analytical solutions of the governing differential equations of the structural members. The object of this paper is to exploit the high stability of root-counting methods by applying them to the conventional linear eigenvalue problem, in order to obtain eigenvalues and eigenvectors with high accuracy.

2. Sign count of dynamic stiffness matrix

The linear eigenvalue problem in structural dynamics can be stated mathematically as attempting to find a positive real parameter λ in the equation

$$\mathbf{K}_D(\lambda) \equiv (\mathbf{K} - \lambda\mathbf{M})\mathbf{x} = \mathbf{0}, \tag{1}$$

so as to make the n-dimensional vector $\mathbf{x}$ non-trivial, where the $n \times n$ dimensional real, symmetric, non-negative definite matrices $\mathbf{K}$ and $\mathbf{M}$ are the static stiffness and mass matrices, respectively. It is well known that there exists a non-singular matrix $\mathbf{X}$ whose ith column is the eigenvector $\mathbf{x}_i$ associated with the eigenvalue λ_i, which satisfies

$$\mathbf{X}^{\mathrm{t}}\mathbf{K}_D(\bar{\lambda})\mathbf{X} = \mathrm{Diag}(\lambda_1 - \bar{\lambda}, \ldots, \lambda_i - \bar{\lambda}, \ldots, \lambda_n - \bar{\lambda}). \tag{2}$$

By virtue of Sylvester's law of inertia [9], if a series of non-singular matrices $\mathbf{P}_1, \ldots, \mathbf{P}_m$ is found such that

$$\mathbf{P}_i^{\mathrm{t}}\mathbf{K}_D(\bar{\lambda})\mathbf{P}_i = \mathbf{D}_i \quad (i = 1, \ldots, m), \tag{3}$$

where each of the $\mathbf{D}_i$ $(i = 1, \ldots, m)$ is a diagonal matrix, then each $\mathbf{D}_i$ has the same number of negative elements, called the sign count of the dynamic stiffness matrix $\mathbf{K}_D(\bar{\lambda})$ and denoted $s\{\mathbf{K}_D(\bar{\lambda})\}$. Moreover, Eq. (2) shows that $s\{\mathbf{K}_D(\bar{\lambda})\}$ equals the number of eigenvalues exceeded by $\bar{\lambda}$.

Knowing $s\{\mathbf{K}_D(\bar{\lambda})\}$ enables bounds on the eigenvalues to be obtained. For example, a trial frequency $\bar{\lambda}_{k1}$ is a lower bound on the kth eigenvalue λ_k if $s\{\mathbf{K}_D(\bar{\lambda}_{k1})\} < k$, or an upper bound if $s\{\mathbf{K}_D(\bar{\lambda}_{k1})\} \geqslant k$. Once lower and upper bounds have been found, convergence on λ_k is achieved by successively evaluating the sign count at trial frequencies lying between the two bounds.

Since only the signs, and not the values, of the elements of the corresponding diagonal matrix are needed when evaluating $s\{\mathbf{K}_D(\bar{\lambda})\}$, the sign count is quite insensitive to numerical errors. It is this feature of the sign count that makes the algorithms based on it almost infallible.

3. Reduction of dynamic stiffness matrix to diagonal

A generalized inner product of two vectors $\mathbf{p}_i$ and $\mathbf{p}_j$ of order n is introduced as

$$(\mathbf{p}_i, \mathbf{p}_j) = \mathbf{p}_i^{\mathrm{t}}\mathbf{K}_D(\bar{\lambda})\mathbf{p}_j. \tag{4}$$

Any set of independent vectors $\mathbf{p}_1^{(1)}, \mathbf{p}_2^{(1)}, \ldots, \mathbf{p}_n^{(1)}$ of order n can be transformed to an orthogonal set $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$, in the spirit of the method of modified Gram–Schmidt orthogonalization, as follows. The first desired vector is simply chosen as $\mathbf{p}_1 = \mathbf{p}_1^{(1)}$. Then, for $k = 2, \ldots, n$, after performing the transformations

$$\mathbf{p}_i^{(k)} = \mathbf{p}_i^{(k-1)} - \frac{\left(\mathbf{p}_{k-1}, \mathbf{p}_i^{(k-1)}\right)}{\left(\mathbf{p}_{k-1}, \mathbf{p}_{k-1}\right)}\mathbf{p}_{k-1} \quad (i = k, \ldots, n), \tag{5}$$

the kth vector is chosen as $\mathbf{p}_k = \mathbf{p}_k^{(k)}$. If $(\mathbf{p}_k^{(k)}, \mathbf{p}_k^{(k)}) = 0$, but $(\mathbf{p}_{k'}^{(k)}, \mathbf{p}_{k'}^{(k)}) \neq 0$ for some $k' > k$, then $\mathbf{p}_{k'}^{(k-1)}$ is exchanged with $\mathbf{p}_k^{(k-1)}$ to make the choice valid.

It can be seen from Eq. (5) that $\mathbf{p}_k$ is a linear combination of $\mathbf{p}_i^{(1)}$ $(i = 1, \ldots, k-1)$. If each $\mathbf{p}_i^{(1)}$ is selected as the ith column of the unitary matrix of order n, then the components of $\mathbf{p}_k$ have the feature that $p_{kk} = 1$ and $p_{kj} = 0$ for $j > k$. The matrix $\mathbf{P}$ whose kth column is $\mathbf{p}_k$ is an upper triangular matrix whose diagonal elements are all unity, and satisfies

$$\mathbf{P}^{\mathrm{t}}\mathbf{K}_D(\bar{\lambda})\mathbf{P} = \mathbf{D}, \tag{6}$$

where $\mathbf{D}$ is a diagonal matrix. Being real and symmetric, the dynamic stiffness matrix can be uniquely decomposed into the form

$$\mathbf{K}_D(\bar{\lambda}) = \mathbf{L}_d\mathbf{D}\mathbf{L}_d^{\mathrm{t}}, \tag{7}$$

where $\mathbf{L}_d$ is a lower triangular matrix. Comparison between Eqs. (6) and (7) gives

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