Electric Power Systems Research 144 (2017) 63–71

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A novel network model for optimal power flow with reactive power and network losses

Zhifang Yang, Haiwang Zhong, Qing Xia*, Chongqing Kang

State Key Lab. of Power System, Dept. of Electrical Engineering, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 12 July 2016
Received in revised form 25 October 2016
Accepted 12 November 2016

Keywords:
Convex relaxation
Network model
Network losses
Optimal power flow (OPF)
Power flow equations
Taylor series expansion

ABSTRACT

To improve the accuracy of the DC network model, we re-examine the power flow equations and propose an improved network model for optimal power flow (OPF) calculation with reactive power and network losses. Voltage angle and the square of voltage magnitude are used as independent variables. A mathematical transformation of the nonlinear voltage magnitude term is used, which decomposes the voltage magnitude term to a linear expression and a quadratic expression. The quadratic expression presents the influence of voltage magnitude on network losses. To handle the non-convexity of the OPF model caused by the network losses, a convex relaxation method is used. The relaxed model and the proposed OPF model are typically equivalent in transmission systems. Methods of restricting the potential relaxation errors are introduced. It is shown that the accuracy of the proposed network model can be further improved with initial points of voltage angles. Case studies in several IEEE benchmark systems validate the performance of the proposed formulation and approach.

1. Introduction

1.1. Literature review

Optimal power flow (OPF) calculations are fundamental to power system planning and scheduling, market clearing and many other applications. Considering the large quantity of electric energy production and consumption around the world, improvements in OPF solutions could result in annual savings of billions of dollars for power industries [1].

The OPF problem was first formulated in the 1960s [2]. It has been proven to be a rather difficult problem to solve, mostly because of the intractability of AC power flow equations. There has been massive research into the development of efficient algorithms to solve the AC OPF models. Conventional methods often involve iterative solving based on an initial guess [3–7]. Successive linear approximation approach is proposed in Refs. [8–10]. There are also efforts that use artificial intelligence techniques to solve AC OPF models [11,12]. Recently, solving the AC OPF problem via convexification approaches attracts research interests. The formulations include *semidefinite relaxation* (SDR) [13–15], *quadratic convex relaxation* (QCR) [16] and *second-order cone relaxation* (SOCR) [17–21]. The solutions from the relaxations always provide a lower bound for the original OPF problem. Nevertheless, it is hard to recover an AC feasible solution when the optimal condition does not hold. The gap between the SDR solution and the AC OPF solution could be up to 30% [16]. SOCR is more computationally efficient than the SDR [22]. However, the SOCR method can result in a significant deficit in the modelling accuracy in a simple radial network [23]. A sufficient condition for the convex relaxation of the SOCR model in distribution networks is provided in Ref. [21]. In Ref. [24], a strong SOCR method is proposed to extend the application of the SOCR to the meshed networks. The accuracy is greatly improved over the traditional SOCR method. Because constraints that ensure the coupling relationship among voltage angles are required for every loop in the network, the computational burden increases in heavily meshed networks.

To solve the OPF model more efficiently and robustly, industries seek approximated network models to reduce the computational burden of the OPF problems. By using such simplified network models, the nonlinearity and nonconvexity of the OPF model are reduced at the expense of a reasonable loss in accuracy. The linear "active power only" DC network model is one of the representatives [25]. In the DC network model, losses and reactive power are completely ignored. As a result, the DC OPF model may lead to less economical and even insecure solutions, especially in stressed systems and systems with strong coupling between active and reactive power [8]. To improve the accuracy of the DC network model,

* Corresponding author. Fax: +86 1062571485.
E-mail address: qingxia@tsinghua.edu.cn (Q. Xia).

http://dx.doi.org/10.1016/j.epsr.2016.11.009
0378-7796/

Nomenclature: Listed below are the main mathematical symbols used throughout this paper for quick reference.

Variables and parameters

g_{ij}/b_{ij}	Conductance/susceptance of branch (*i*,*j*)
G_{ij}/B_{ij}	Real/imaginary part of Y_{ij}
N	Number of buses
$P_{d,i}/Q_{d,i}$	Active/reactive load consumption at bus *i*
P_g/Q_g	Generator active/reactive power production
P_i/Q_i	Active/reactive power injections at bus *i*
P_i^L/Q_i^L	Active/reactive network losses allocated to bus *i*
P_{ij}/Q_{ij}	Active/reactive power flows on branch (*i*,*j*)
v_i	Voltage magnitude at bus *i*
$v_{ij,s}^s$	$v_{ij,s}^s = \left(v_i^2 - v_j^2\right)^2$
Y_{ij}	The (i, j) entry of the admittance matrix Y
θ_{ij}	Voltage angle difference between buses *i* and *j*
$\theta_{ij,s}$	$\theta_{ij,s} = \theta_{ij}^2$
$\gamma, w, \alpha, \beta, \zeta, \kappa, \rho, \pi, \vartheta, \tau, \mu, \varphi, \sigma$	Dual variables

Vectors and matrices

$\mathbf{P}_g/\mathbf{Q}_g$	Vector of generator active/reactive power production
$\mathbf{v}_m/\boldsymbol{\theta}$	Vector of voltage magnitude/angle
$\mathbf{v}^s$	$[v_1^2, v_2^2, \ldots, v_N^2]$

Sets

$\mathcal{N}, \mathcal{K}, \mathcal{G}$	Sets of buses, branches, and generators

network losses and reactive power need to be modelled. In existing research, losses are usually expressed as a function of θ^2 in the DC network model [26–29]. Methods that handle the non-convexity caused by the quadratic loss terms include the convex relaxation method [26] and the piecewise linearization method [27–29]. The accuracy of the modelling of losses still has room to improve, because the influence of voltage magnitude on losses has not been included. For reactive power, there are methods proposed that incorporate the reactive power and voltage magnitude in the DC network model via analysing the Taylor series expansion. In Ref. [28], an improved network model is obtained. Except for quadratic loss terms, the network model is linear with voltage angles and off-nominal voltage magnitude as variables. Piecewise linear approximation is used to handle the quadratic loss term. Integer variables are introduced to avoid the approximation errors. Convex relaxation approach is discussed as well and the physical insight of the relaxation is provided. The network model described above is used for transmission planning problems in Ref. [29]. For the network models used in Refs. [28] and [29], the accuracy of the voltage magnitude terms can be further improved. Because the first-order Taylor series expansion of the voltage magnitude terms is used in the network model, the influence of voltage magnitude on network losses is ignored. The accuracy of the approximation for the nonlinear voltage magnitude terms is crucial for accurately modelling the reactive power, voltage magnitude, and network losses. In this paper, we use a mathematical transformation to provide a better approximation for voltage magnitude terms.

1.2. Contributions

For existing approximated network models, there is still room to improve, especially for the modelling of the voltage magnitude and network losses. To address these aspects, an improved network model for OPF with consideration of reactive power and network losses is proposed. This paper provides following contributions:

1) A novel network model with reactive power and network losses is proposed. To take advantage of the quasi-linear relationship of *P*-θ, the polar coordinate power flow equations is used. To improve the accuracy of the modelling of voltage magnitude and reactive power, a mathematical transformation for the nonlinear voltage magnitude term is used. The voltage magnitude term is decomposed into a linear term that reflects the influence of voltage magnitude on the power distribution and a quadratic term that reflects the influence of voltage magnitude on network losses. Case studies confirm that the proposed network model is more accurate than several existing approximated network models, including the network models used in Refs. [26] and [27] and the network models used in Refs. [28] and [29].
2) A convex relaxation method is used to handle the non-convexity brought to the proposed OPF model by the loss terms. The physical insight of the relaxation is illustrated. The relaxation model and the proposed OPF model are equivalent under certain conditions, which are typically satisfied for transmission systems. Cuts are added in case the conditions do not hold.
3) A warm-start network model using the initial point of voltage angles is proposed, which can further improve the performance of the OPF model. Due to the quasi-linear relationship of *P*-θ in transmission systems, a high-quality initial point of voltage angles is easily accessible (for example, use the DC OPF method). The proposed warm-start method provides a guidance for formulating more accurate network models using initial points of voltage angles.

2. Derivation of the proposed network model

In this section, the power flow equations are analysed from the original form. The obtained network model will be the basis of the proposed OPF method.

The power flow equations are as follows:

$$P_i(\boldsymbol{v_m}, \boldsymbol{\theta}) = \sum_{j=1}^{N} (v_i v_j G_{ij} \cos\theta_{ij} + v_i v_j B_{ij} \sin\theta_{ij}) \tag{1}$$

$$Q_i(\boldsymbol{v_m}, \boldsymbol{\theta}) = -\sum_{j=1}^{N} (v_i v_j B_{ij} \cos\theta_{ij} - v_i v_j G_{ij} \sin\theta_{ij}) \tag{2}$$

The branch flow expressions are as follows:

$$P_{ij}(\boldsymbol{v_m}, \boldsymbol{\theta}) = (v_i^2 - v_i v_j \cos\theta_{ij}) g_{ij} - v_i v_j b_{ij} \sin\theta_{ij} \tag{3}$$

$$Q_{ij}(\boldsymbol{v_m}, \boldsymbol{\theta}) = -(v_i^2 - v_i v_j \cos\theta_{ij}) b_{ij} - v_i v_j g_{ij} \sin\theta_{ij} \tag{4}$$

To simplify the power flow equations, the second-order approximations of the sine and cosine functions are used:

$$\sin\theta_{ij} \approx \theta_{ij}, \cos\theta_{ij} \approx 1 - \frac{\theta_{ij}^2}{2} \tag{5}$$

Substituting (5) into (1) and (2), the following expressions can be obtained:

$$P_i = \sum_{j=1}^{N} G_{ij} v_i v_j + \sum_{j=1}^{N} B_{ij} v_i v_j \theta_{ij} + \sum_{j=1}^{N} (-G_{ij}) v_i v_j \frac{\theta_{ij}^2}{2} \tag{6}$$

$$Q_i = -\sum_{j=1}^{N} B_{ij} v_i v_j + \sum_{j=1}^{N} G_{ij} v_i v_j \theta_{ij} + \sum_{j=1}^{N} B_{ij} v_i v_j \frac{\theta_{ij}^2}{2} \tag{7}$$

In (6) and (7), $v_i v_j$ and θ_{ij} are tightly coupled in the last two terms. Regarding $v_i v_j$ as a whole, the first-order Taylor series expansions of