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Interval power flow analysis via multi-stage affine arithmetic for unbalanced distribution network

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ABSTRACT

Integrating large numbers of intermittent energy resources, such as photovoltaic generations and wind turbines, brings huge uncertainty to the operation of distribution network. In this paper, an interval power flow method via multi-stage affine arithmetic is proposed to address the impact of distributed generation output power and load uncertainty on the power flow solution for the unbalanced distribution network, which acquires affine expression of bus voltage by solving the fixed component, first-order perturbation and non-linear perturbation of the bus complex voltage respectively. Multi-numerical results based on modified IEEE 123 nodes test feeder are presented and discussed, demonstrating that the proposed method achieves much better performance than existing ones in both precision and computation aspects.

1. Introduction

Power flow analysis is one of the most fundamental and most frequently-used tools in power engineering, such as power network optimization [1], voltage/var control [2], power system state estimation [3], etc. In recent years, with the increase of distributed generations (DGs) penetration in distribution network, the traditional "passive network" is turning into the so-called "active distribution network" (ADN), which possesses more schedulable energy resources and more flexible operation modes [4], putting forward higher requirements for online power flow analysis [5,6]. However, huge uncertainty is introduced into power distribution system operation and analysis due to the connection of large quantities of DGs, like photovoltaic panels (PVs) and wind turbines (WTs), of which the output power is intermittent and stochastic with the continuous variation of weather conditions [6,7]. Deterministic power flow algorithms cannot describe the influence of uncertainty and cannot satisfy the requirement of ADN power flow analysis [8].

Many indeterministic methods have been developed to study the uncertainty of power flow introduced by DGs, mainly including probabilistic power flow methods [9,10] and interval power flow methods [11–13]. When calculating probabilistic power flow, the probability distribution function (PDF) of each DG output power is estimated firstly, then probability distribution of the system power flow is acquired via convolution. Unfortunately, exact PDF of DG output power is hard to estimate, which limits its application [14]. What's worse, convolution is computationally cumbersome for online analysis of the distribution network. On the contrary, the model of interval power flow is rather simple and the solution is much more intuitionistic, which exports the upper bound and lower bound of the bus voltage if power injection interval of each bus is given [15]. When implementing interval power flow analysis, the uncertain output power of each DG or load power is expressed as an interval number [16], then non-linear power flow equations including interval numbers are solved according to interval analysis theory [17]. Interval arithmetic (IA) is firstly introduced into power flow analysis in Ref. [15] to study the influence of parameter uncertainty. In Refs. [18] and [19] the Krawczyk–Moore iteration method is proposed to solve the interval power flow equations respectively. The interval fast decoupled algorithm is studied in Ref. [21], which acquires voltage magnitude interval and voltage angle interval by solving two groups of interval linear equations respectively. In Ref. [12], an interval forward-backward sweep method is developed for balanced radial distribution network power flow analysis considering the load uncertainty, while Ref. [20] accommodates the uncertainty of wind turbine power by the same method. Ref. [22] further generalizes the interval forward-backward sweep method to unbalanced system, which is common in low voltage distribution networks. Interval arithmetic (IA) has been widely accepted as an

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effective approach to addressing uncertainty, but it always makes excessively pessimistic estimates to the fluctuation range of system power flow. To reduce the conservatism of interval arithmetic solutions, the interval power flow models are transformed to interval optimization problems, which solve the lower and upper bound of bus voltage and a more accurate interval solution is acquired [23,24]. Essentially, interval arithmetic ignores the correlations among interval variables [16] and will expand the range of actual solution inevitably. Consequently, interval variables are presented as affine forms in Refs. [25] and [26], and then power flow equations with affine variables are solved by the Newton type method and the forward-backward sweep method. Refs. [27] and [28] generalize the forward–backward sweep method based on affine arithmetic (AA) to three phase unbalanced systems, and an index of relative influence of uncertain variables on outcomes is proposed in Ref. [28] for quantifying the impacts of individual uncertain factors on power flows and bus voltages. Compared to IA, AA could keep track of the linear correlation among different uncertain variables, thus reducing the solution conservatism.

Power flow equations are a group of non-linear equations, and solving the system iteratively inevitably requires many non-linear operations. As a result, when solving the interval power flow model via AA, large numbers of non-linear affine operations are inevitable. To simplify power flow analysis, Ref. [29] proposes an approximated power flow models for unbalanced distribution network, which approximates the power flow equations into a group of linear equations concerning the real and imaginary components of bus voltage respectively.

By AA, linear correlations among different affine variables are kept accurately, but when non-linear affine operations are done, new affine variables are introduced approximately to keep the outcome as affine combinations. Consequently, when adopting AA, large numbers of non-linear affine operations will introduce considerable errors during power flow iterations. Also, all aforementioned methods including IA and AA, can just give an pessimistic estimation of the solution hull, but no quantitative mapping from input variables (the uncertain power injections) to output variables (the uncertain bus complex voltages) is given. To solve the aforementioned problems, a multi-stage affine arithmetic (MSAA) method based on the approximated power flow equations in Ref. [29] is proposed in this paper for unbalanced distribution network interval power flow analysis. It should be pointed out that in Ref. [29], power system loads are treated as voltage dependent loads while in this study, only constant power loads are considered to accommodate uncertainty. The main contributions of this paper are as follows:

(1) A MSAA based interval power flow algorithms is proposed, which reduces interval expansion level introduced by non-linear affine operations and accelerates convergence of interval power flow iterations.
(2) The multi-phase model is adopted considering the load and DG unbalance and line parameters asymmetry widely existing in low voltage distribution network.

The rest of the paper is organized as follows. Section 2 gives a simple introduction to AA and the approximated power flow equations. Section 3 presents the proposed MSAA based interval power flow method. Multi-cases study based on modified IEEE 123 buses unbalanced distribution network by the proposed MSAA are implemented and compared with AA, IA, Monte-Carlo simulation (MCS) methods in Section 4, which demonstrates the proposed method. And Section 5 concludes this paper.

2. Affine arithmetic and linear power flow equations

2.1. Affine arithmetic

Affine arithmetic was proposed by Comba and Stolfi in Ref. [30] to solve the computer mapping problem. Affine variables could represent uncertainty as well as keep track of correlations among different variables.

In affine arithmetic an interval number is represented as:

$$\hat{x} = x_0 + x_1\varepsilon_1 + \cdots + x_n\varepsilon_n \tag{1}$$

where x_0 is the central value just like that of an interval number; x_i is the perturbation coefficient; ε_i is the noise component locating in $[-1,1]$, which represents each independent uncertain source of x.

For two arbitrary real or complex affine numbers $\hat{x}$ and $\hat{y}$, operation rules of plus, minus, multiplication, etc. are defined as (2)–(6):

$$\hat{x} \pm \hat{y} = (x_0 \pm y_0) + (x_1 \pm y_1)\varepsilon_1 + \cdots + (x_n \pm y_n)\varepsilon_n \tag{2}$$

$$\alpha\hat{x} = \alpha x_0 + \alpha\sum_{i=1}^{n} x_i\varepsilon_i \tag{3}$$

$$\hat{x} \pm \alpha = (x_0 \pm \alpha) + \sum_{i=1}^{n} x_i\varepsilon_i \tag{4}$$

$$\left(\hat{x}\right)^* = x_0^* + \sum_{i=1}^{n} x_i^*\varepsilon_i \tag{5}$$

$$\hat{x} \times \hat{y} = x_0y_0 + \sum_{i=1}^{n}(x_0y_i + y_0x_i)\varepsilon_i + z_{error}\varepsilon_{error} \tag{6}$$

where z_{error} is the approximated error, ε_{error} is a newly introduced noise component by the multiplication; $()^*$ represents conjugate operation.

Real affine division operation $\hat{x}/\hat{y}$ is equivalent to calculating $\hat{x} \times 1/\hat{y}$, and transforming $\hat{y}$ to an interval number $[a, b]$, then:

$$\frac{1}{\hat{y}} = \alpha\left(y_0 + \sum_{i=1}^{n} y_i\varepsilon_i\right) + \beta + \gamma\varepsilon_{err} \tag{7}$$

where $\alpha = -\frac{1}{ab}$, $\beta = \frac{a+b+2\sqrt{ab}}{2ab}$, $\gamma = \frac{a+b-2\sqrt{ab}}{2ab}$ and ε_{err} is a newly introduced noise component by reciprocal operation.

Complex affine reciprocal operation is usually transformed to a real reciprocal operation by:

$$\frac{1}{\hat{y}} = \left(\hat{y}\right)^* \times \frac{1}{\hat{y}\left(\hat{y}\right)^*} \tag{8}$$

More details about affine arithmetic and its derivations can be found in Refs. [31,32].

2.2. Approximated linear power flow [29]

For an either balanced or unbalanced distribution network with N nodes, the power flow equations are:

$$\mathbf{I} = \mathbf{Y}V \tag{9}$$

$$\mathbf{I} = \begin{pmatrix} \mathbf{I}_S \\ \mathbf{I}_N \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{SS} & \mathbf{Y}_{SN} \\ \mathbf{Y}_{NS} & \mathbf{Y}_{NN} \end{pmatrix}, \mathbf{V} = \begin{pmatrix} \mathbf{V}_S \\ \mathbf{V}_N \end{pmatrix} \tag{10}$$

where $\mathbf{Y}$ is the admittance matrix; $\mathbf{I}$, $\mathbf{V}$ represent bus current injections and bus voltage vector respectively, and more details about the unbalanced power flow equations can be seen in Ref. [33]; subscript S represents the slack bus, and N represents the others.