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Inelastic analysis of 2D solids using a weak-form RPIM based on deformation theory

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Abstract

A radial point interpolation method (RPIM) is presented for inelastic analysis of 2D problems. In this method, the problem domain is represented by a set of scattered nodes and the field variable is interpolated using the field values of local supporting nodes. Radial basis function (RBF) augmented with polynomials is used to construct the shape functions. These shape functions possess delta function property that makes the numerical procedure very efficient and many techniques used in the finite element method can be adopted easily. Galerkin weak-form formulation is applied to derive the discrete governing equations and integration is performed using Gauss quadrature and stabilized nodal integration. The pseudo-elastic method proposed by H. Jahed et al. is employed for the determination of stress field and Hencky's total deformation theory is used to define the effective material parameters. Treated as field variables, these parameters are functions of the final state of stress fields that can be obtained in an iterative manner based on pseudo-linear elastic analysis from one-dimensional uniaxial material curve. A very long thick-walled cylinder and a V-notched tension specimen are analyzed and their stress distributions match well with those obtained by finite element commercial software ANSYS or the available literature.

Keywords: Meshfree method; Material nonlinearity; Stress distribution, Point interpolation; Radial basis function (RBF)

1. Introduction

Nonlinearities arise from several sources in engineering problems. For example, a nonlinear material response can result from elasto-plastic material behavior in which the relations between stresses and strains are complicated functions or from hypo-/hyper-elastic effects of certain forms when large strain or large deformation occurs in solids and structures. From the viewpoint of mathematics, the discretization of nonlinear problems results a set of simultaneous equations, in which equation coefficients depend on the solutions or their derivatives. Due to the mathematical difficulties, analytical solutions are limited to problems with very simple geometries and external loadings. Numerical techniques are commonly used in dealing with problems with material or geometric nonlinearities. Finite element method is firmly accepted as a most powerful general tool for numerical solution of a variety of nonlinear problems encountered in engineering [1,2]. The analysis of such problems normally proceeds in an incremental manner since the solution at any stage may not only depend on current state but also on the precious loading history. Finite element implementation of elasto-plastic problems based on flow theory of plasticity was detailed by Owen and Hilton [3].

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Another way for inelastic analysis is based on elastic solutions. The idea of using elastic solutions for approximation of inelastic behavior has a long history. Neuber obtained elasto-plastic stresses and strains at concentration point using elastic solutions in early 1960s [4]. Dhalla and Jones used finite element elastic analysis to predict the limit loads [5]. Seshadri et al. [6] then modified and improved this method and named it generalized local stress and strain (GLOSS) method. Recently, Jahed et al. [7] developed a comprehensive method for inelastic analysis based on elastic solutions. In the method material constants are treated as spatial variables, which are updated in an iterative procedure. In their work, cylindrical pressure vessel was considered as an assembly of finite number of strips and closed form of elastic solution was used within each strip. The scheme based on the total strain energy provides the most rapid convergence in an iterative procedure and hence inelastic analytical solution can be readily obtained for thick-walled pressure cylinders. Desikan and Sethuraman [8] extended the idea to finite element method for the determination of inelastic whole field solutions. In their work, material nonlinear problem could be solved using the proposed pseudo-elastic linear finite element method with suitable updating of elastic material properties during the process of iteration.

In recent years, meshless or meshfree methods have been promising and attractive for solving boundary value or initial value problems, especially for cases with large deformation, crack propagation or movement of a free surface since nodes are no longer bonded with specific elements and they can be freely added or removed when necessary. Several meshfree methods have been successfully developed and applied to varieties of engineering problems, among which are some famous ones such as smooth particle hydrodynamics (SPH) [9,10], element-free Galerkin (EFG) method [11,12], reproducing kernel particle method (RKPM) [13], meshless local Petrov–Galerkin method (MLPG) [14], just to name only a few here. Point interpolation method (PIM) based on weak-form Galerkin or Petrov–Galerkin method was originally proposed by Liu et al. and it has been successfully applied to many engineering problems [12,15–18]. In this method, field variable is exactly interpolated using the field nodal values in local support domain by means of radial basis function (RBF) augmented with polynomial terms. The generated shape functions possess delta function properties and the essential boundary conditions can, therefore, be conveniently imposed as in conventional FEM. The procedure for construction of shape functions is more efficient and straightforward than moving least squares (MLS) approach. Recently, PIM and MLS based on a combined formulation using both weak- and strong-form formulations were developed, or in short meshfree weak–strong form method and successfully applied to solid and fluid problems [19,20].

The objective of this work is to develop a point interpolation meshfree method for solving inelastic problems based on Hencky's deformation theory. In the present formulation, the material constants like Young's modulus and Poisson's ratio are treated as field variables and iteration procedure is used to update these variables until all the equivalent stress–strain points coincide with the uniaxial experimental material curve. The projection method proposed by Jahed et al. [7] will be employed to calculate these effective material parameters. For easy implementation, triangular background cells are constructed and cell-based nodal selection scheme is recommended. Some problems using von-Mises materials with/without work hardening, or materials following Ramberg–Osgood formula will be analyzed to demonstrate the convergence and stability of the present method.

2. Radial point interpolation method (RPIM)

Consider a field function $u(\mathbf{x})$ defined by a set of arbitrary distributed nodes $\mathbf{x}_i$ $(i = 1,2,\ldots,N)$ in a domain Ω with boundary Γ. It is assumed that only the surrounding nodes near a point of interest, $\mathbf{x}_Q$, have effect on $u(\mathbf{x}_Q)$. The domain covering these nodes is called the support domain of $\mathbf{x}_Q$.

Using n nodes in the local support domain of point $\mathbf{x}_Q$, the RBF augmented with polynomial terms interpolates the field variable $u(\mathbf{x})$ in form of

$$u^h(\mathbf{x},\mathbf{x}_Q) = \sum_{i=1}^{n} r_i(\mathbf{x})a_i + \sum_{j=1}^{m} p_j(\mathbf{x})b_j = \mathbf{r}^{\mathrm{T}}(\mathbf{x})\mathbf{a} + \mathbf{p}^{\mathrm{T}}(\mathbf{x})\mathbf{b}, \quad (1)$$

where a_i and b_j are the unknown coefficients for the radial basis $r_i(\mathbf{x})$ and the polynomial basis $p_j(\mathbf{x})$, respectively. For example, a basis in two-dimensional problems is provided by $\mathbf{p}^{\mathrm{T}} = \{1, x, y, x^2, xy, y^2\}$ for $m = 6$. The numbers of the radial basis n and the polynomial basis m are chosen based on the reproduction requirement. Several radial basis functions (RBFs) are available for RPIM [12]. Among them, multi-quadric (MQ) function is very stable in practical computations, which is expressed in form of

$$r_i(x,y) = [(x-x_i)^2 + (y-y_i)^2 + Cd_{\min}^2]^q. \quad (2)$$

Here, C and q are shape parameters, which have great effects on final solutions when Gauss integration is used for weak form and they will be discussed in numerical examples. The coefficients a_i and b_j are determined by enforcing the Eq. (1) pass through n data nodes in the support domain. Therefore the following equations can be obtained