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A Feedback Optimal Control by Hamilton-Jacobi-Bellman Equation

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Abstract

This paper presents a computational method to deal with the Hamilton-Jacobi-Bellman equation with respect to a nonlinear optimal control problem. With Bellman dynamic programming principle and the nonlinear minimization method, the feedback optimal control is obtained by means of the value function under certain smooth assumptions. The main work is to present a global minimizer flow in the Hamilton-Jacobi-Bellman equation with an iteration process for solving the corresponding difference equation.

Keywords: Hamilton-Jacobi-Bellman equation; Feedback optimal control; Nonlinear minimization; Global minimizer flow; Difference equation

1. Primal Problem.

In this paper, the notation $\|.\|$ represents a norm for the specified space concerned. The primal goal of this paper is to present a numerical solution of the Hamilton-Jacobi-Bellman equation with respect to the following nonlinear optimal control problem(primal problem $(\mathcal{P})$ in short).

$$(\mathcal{P})\ \ \min J(0,x_0,u)=\int_0^T[F(x)+P(u)]dt \tag{1.1}$$

$$s.t.\ \dot{x}=f(x)+g(x)u,\ \ x(0)=x_0,\ t\in[0,T],\ x\in R^n,\ u\in R^m, \tag{1.2}$$

where $P(u)$ is twice continuously differentiable on R^m and $\nabla^2 P(u)>0, \forall u\in R^m$. We assume that

$$\liminf_{\|u\|\to\infty}\frac{P(u)}{\|u\|^2}>0. \tag{1.3}$$

In the control system, $F(x), f(x), g(x)$ are continuously differentiable on R^n and x_0 is a given vector in R^n.

If $P(u)$ is a positive definite quadratic form with respect to u and $F(x)$ is a positive semi-definite quadratic form with respect to x while $f(x)+g(x)u$ is linear with respect to (x,u), then the problem $(\mathcal{P})$ is a classical linear-quadratic optimal control problem([1],[2]). On the theoretic aspect, it is difficult to find out an optimal control to a nonlinear problem. Traditionally, in most works on nonlinear optimal control, the authors focus on either numerical solution to Hamilton-Jacobi-Bellman equations([3]) or theoretical study concerning Pontryagin maximum principle([4]). Here we give a little bit of literature review on the paper [3]. In [3], with introducing a modified method of characteristics procedure, the authors studied a numerical viscosity solution to the Hamilton-Jacobi-Bellman