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Numerical Discretization of Boundary Control Problems for Systems of Balance Laws: Feedback Stabilization

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Abstract

We consider the boundary control of discretized problems based on systems of linear conservation laws including source terms. For a discrete Lyapunov function we prove the exponential decay in L^2-norm using first-order finite volume methods. We give explicit computations of the decay rates and apply the proposed framework to the numerical boundary feedback stabilization of electric transmission lines given by the telegraphers equation.

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Keywords. Systems of balance laws, Lyapunov function, feedback stabilization.

1 Introduction

Boundary control of conservation laws has become a broad subject during the last few years and is an active area for current research. A variety of literature exists on boundary control of hyperbolic systems, in particular analytical results on stability are provided in [7, 8, 11, 12, 16, 22, 23, 25, 26]. For a comprehensive overview we refer to [3, 9] and the references therein. Further analytical results also include hyperbolic systems with source terms (so-called hyperbolic balance laws) for special applications such as gas dynamics [2, 18], water flow in open canals [13, 19] or electric transmission lines [15]. Furthermore, it is proven for the analytical case that feedback boundary values, designed under certain conditions, yield exponential decay of a continuous Lyapunov function [3, 10, 12].

However, the focus of this paper is to perform a numerical stability analysis of linear hyperbolic systems of conservation laws including source terms. Therefore we discretize the system by a first-order finite volume scheme combined with a splitting method to solve the source term problem. Inspired by [1], we intend to answer the question under which conditions a numerical scheme yields an exponential decay of the discrete solution to the hyperbolic system in L^2-norm. These conditions obviously depend on the proposed numerical scheme and can be explicitly given, in contrast to many theoretical results.

Also in [1] the authors present some proofs concerning under which conditions an exponential decay of a discrete solution to a hyperbolic conservation law can be observed. However, the latter investigations only consider systems without a source term. Therefore, the conditions for exponential decay of the discrete solution to hyperbolic systems including source terms need to be established.

In most analytical results, as [3, 10, 12] for example, the decay rates cannot be formulated explicitly. In this work, we give an explicit representation of the decay rates and show its numerical convergence behavior. Furthermore, we design upper bounds for the feedback boundary values, with the help of a discrete Lyapunov function and also use this Lyapunov function to design linear feedback boundary values.

All questions are first investigated in the context of systems of linear hyperbolic conservation laws with a (nonlinear) source term. The results are then applied to a special class of hyperbolic

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