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Abstract

The aim of this paper is to study the problem of stabilization of nonlinear systems with a quadratic control. Under the assumption that a control Lyapunov function (CLF) is known, we derive necessary and sufficient conditions for the stabilization by continuous feedback controls explicitly computed. Illustrative examples are presented.

Keywords: Quadratic control systems, Control Lyapunov function, Global stabilization.
2010 MSC: 93C10, 93D15

1. Introduction

The stabilization of nonlinear systems has been extensively investigated by many authors, and various techniques have been developed to design stabilizing feedback (see [2, 4, 5, 6, 7, 10, 11] and the references therein). Most of the results are classically obtained by a Lyapunov approach. In practice, this approach consists in finding a feedback law together with a positive definite function which decreases along the trajectories of the closed-loop system. It is not always easy to find such a function, but Artstein [1] introduced the notion of control Lyapunov function (CLF) and, knowing this function, he showed how it is possible to design a stabilizing feedback law.

This technique has been widely adopted in the past decades by many authors (see [3, 9, 10, 11] and the references therein). These authors provide sufficient conditions for the existence of almost continuous stabilizing feedback laws, that are continuous everywhere but, possibly, not at the origin. In [1], Artstein pointed out that for nonlinear systems which are affine in the control, the stabilizability (i.e. the existence of a stabilizing feedback) is equivalent to the existence of a CLF. Moreover, as quoted by Sontag in [9], the Lyapunov function as well as the feedback can be chosen smooth. An explicit and simple proof of Artstein's Theorem [1] is given by Sontag in [9] and revisited later on by Lin and Sontag in [3].

In the proof given in [9], Sontag uses a universal formula for affine systems, but the problem of stabilization of more general nonaffine systems has not yet a universal construction and is an active research field.

In this paper, we consider systems of the form

$$\dot{x} = f_0(x) + uf_1(x) + u^2 f_2(x), \tag{1.1}$$

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