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PII: S0947-3580(16)30098-X
DOI: 10.1016/j.ejcon.2017.04.004
Reference: EJCON 205

To appear in: *European Journal of Control*

Received date: 16 August 2016
Revised date: 27 March 2017
Accepted date: 20 April 2017

Please cite this article as: Chunyu Shen , Lixin Tian , Optimal control and optimality condition of the Camassa-Holm equation, *European Journal of Control* (2017), doi: 10.1016/j.ejcon.2017.04.004

Optimal control and optimality condition of the Camassa-Holm equation

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Abstract

This paper is devoted to the optimal distributed control problem described by Camassa-Holm equation. We firstly investigate the existence and uniqueness of weak solution for the controlled system with appropriate initial value and boundary condition. By contrast with our previous result, the proof without considering viscous coefficient is a bigger improvement. Secondly, based on the well-posedness result, we find a unique optimal control for the controlled system with the quadratic cost functional. Moreover, by means of the optimal control theory, we establish the sufficient and necessary optimality condition of an optimal control, which is another major novelty of this paper. Finally, we also present the optimality conditions corresponding to two physical meaningful distributive observation cases and give an illustration about how to numerically apply the obtained results.

Keywords

Weak solution, Existence and Uniqueness, Optimal control, Sufficient and Necessary optimality condition, Camassa-Holm equation

1. Introduction

The Camassa-Holm (CH) equation, written in the familiar form

$$u_t + 2\kappa u_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad t > 0, \quad x \in R, \tag{1.1}$$

which describes one-dimensional surface waves at a free surface of shallow water under the influence of gravity. The function $u = u(t, x)$ represents the fluid velocity at time t and position x (or equivalently, the height of the water's free surface over a flat bottom), and constant κ related to the critical shallow water wave speed. The CH equation was first reported as an abstract bi-Hamiltonian partial differential equation with an infinitely number of conservation laws by the method of recursion operators [23], but importantly, it came to prominence in the work of Camassa and Holm [11] and Camassa et al. [12], where some of its soliton-type characteristics are described. The solitary waves are smooth if $\kappa > 0$ and peaked in the limiting case $\kappa = 0$. Of physical interest are solutions of Eq. (1.1) which decay at infinity [31]. Eq. (1.1) was subsequently obtained as a model for wave propagation in cylindrical

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