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Infinite horizon optimal control problem of mean-field backward stochastic delay differential equation under partial information☆

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ABSTRACT

This paper investigates an optimal control of an infinite horizon system governed by mean-field backward stochastic differential equation with delay and partial information. Firstly, we establish the existence and uniqueness results for a mean-field backward stochastic differential equation (BSDE) with average delay. Then a class of mean-field time-advanced stochastic differential equations (ASDEs) is introduced as the adjoint equations via duality relation. Meanwhile, necessary and sufficient conditions for optimal control under partial information on infinite horizon are derived. Finally, we apply the theoretical results to study linear-quadratic control problem on infinite horizon to obtain the optimal control, which is explicitly expressed by the solution of a mean-field forward-backward stochastic differential filtering equation.

1. Introduction

Stochastic differential equations involving a large number of interacting particles have attracted increasing attention in the stochastic control theory. The history of the mean-field models can trace their roots to the early works of [15,24]. Recently, Lasry and Lions [18] introduced a mathematical mean-field approach for high dimensional systems that involve a large number of particles. Since then, a lot of works have been done in mean-field problems especially in optimal control, stochastic games and mathematical finance. Interested readers can refer to [5,7,19,20,26] for the stochastic maximum principle under the mean-field models.

Since the work of Pardoux and Peng [31], the theory of (BSDEs) has been studied systematically, see, e.g., the references Barles, Buckdahn and Pardoux [6], Kobylanski [16] and Ma and Liu [22,23], Pardoux and Peng [30], etc. Due to the features that the terminal other than the initial condition is given and a pair of adapted solutions is admitted under some conditions, BSDEs are essentially different from forward stochastic differential equations (SDEs). Intuitively speaking, the adjoint equation of a controlled state process driven by the mean-field SDE is a mean-field BSDE. In 2009, Buckdahn, Djehiche, Li and Peng [8] studied a mean field BSDE in a purely stochastic approach driven by a forward SDE of McKean-Vlasov type. Buckdahn, Li and Peng [9] deepened the investigation of such mean-field BSDEs by studying them in a Markovian framework. They also proved that this mean-field BSDE gave the viscosity solution of a nonlocal PDE.

Stochastic systems with time delay character are common seen in many areas such as epidemiology, engineering, risk management, etc. See, for example, [27,28]. In contrast with standard stochastic control problems, the systems with delay evolve according to not only their current state but also essentially their previous information. Stochastic control problems for systems with delay have also attracted many researchers' attention since the initial work of Kolmanovskiĭ and Maĭzenberg [17], where a linear delayed system with a quadratic cost functional was considered. In 2010, [12] introduced a class of BSDEs with time-delayed generators and proved existence and uniqueness of a solution for a sufficiently

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small time horizon or for a sufficiently small Lipschitz constant of a generator. Very recently, by means of ASDEs, [34] obtained necessary and sufficient conditions of optimality for a backward stochastic delay differential equation (BSDDE) under partial information. They also studied linear-quadratic optimal control problem and gave us an explicit expression of the optimal control in the form of the filtering of adjoint process. On the other hand, BSDDE can also arise in variable annuities, unit-linked products, participating contract and are also useful tools to solve financial problems like option pricing, hedging and portfolio management, which were provided in [11].

However, all the papers mentioned above deal with finite horizon systems. The motivation of studying the infinite horizon optimal control problems arises primarily from the economic and biological sciences where models of this type arise naturally. In 1928, [3] was the first which considered a dynamic optimization model defined on an infinite time horizon. Indeed, any bound placed on the time horizon is artificial when one considers the evolution of the state of an economy of species. For the infinite horizon BSDE, the reader may consult [29,32,37] and references therein. When it comes to a maximum principle, the natural transversality condition in the infinite case would be a zero limit condition. But this property is not necessarily verified. In fact Halkin [14] provided a counterexample for a natural extension of the finite horizon transversality conditions. Thus some care is needed in the infinite horizon case. It is worth mentioning that [13] was the first which required a limit inequality on the terminal condition and illustrated that the infinite horizon case could not be deduced from the finite horizon case. They also proved maximum principles and existence and uniqueness of BSDEs control with infinite horizon, but without delay. Agram, Haadem, Øksendal and Proske [1] established first and second sufficient stochastic maximum principles as well as necessary conditions for the stochastic delay system on infinite horizon with jumps. For the mean-field case, Agram and Røse [4] studied the existence and uniqueness of a solution of the mean-field backward stochastic differential delay equation (MF-BSDDE), where the average delay is not contained. However, we think the inequality $e^{-\beta r}\int_{t+r}^{T+r}e^{\beta u}|\bar{Y}(u)|^2du \le e^{-\beta r}\int_{t}^{T+r}e^{\beta s}|\bar{Y}(s)|^2ds$ given by [4] in Theorem 8 may not be suitable for $r\in[-\delta,0]$. We have not found the condition from which the inequality holds. Besides, what we should pay special attention to is that the MF-BSDDE in this paper differs from the classical BSDE (see e.g. Pardoux and Peng [31]), the infinite horizon BSDE (see e.g. Agram and Øksendal [2]) and the anticipated BSDE (see e.g. Peng and Yang [33]), which is the duality of stochastic differential delayed equation (see e.g. Chen and Wu [10]). Moreover, when only the average and pointwise delays are involved in the state process, however, the existence and uniqueness of solution of MF-BSDDE and optimal control problems are founded to be solvable under certain conditions. The readers can refer to [25] and [35] for the finite horizon case. Therefore, due to these, it is highly desirable to study optimization problems of MF-BSDDE with both average and pointwise delays on infinite horizon.

To the best of our knowledge, there are few papers related to MF-BSDDE on infinite horizon. Specially, [3] studied infinite horizon optimal control of forward-backward mean-field stochastic delayed systems, where only the forward delayed process $\mathbf{X(t)} := (\int_{-\delta}^{0}X(t+s)\mu_1(ds),\dots,\int_{-\delta}^{0}X(t+s)\mu_N(ds))$ is in the state variable and the performance functional. In this paper, we consider a stochastic optimal control problem of mean-field delayed system under partial information, where the state process is governed by a MF-BSDDE. Under the Lipschitz condition, we prove the existence and uniqueness of a solution to the mean-field BSDE with average delay on infinite horizon. To develop our maximum principle, a mean-field ASDE is introduced as the adjoint equation via duality relation. Then under some concavity and transversality conditions, the sufficient condition is also proved. Our paper is different from a recently published paper, Wu and Wang [36] on the similar topic, in the following aspects. First, our control system incorporates both average and pointwise delays in the state equation, which is more general than [36]. Second, we study MF-BSDDE optimal control problem on infinite horizon, which is a continuation of [36] while the finite horizon case is considered. Third, we also prove the existence and uniqueness results of mean-field BSDE with average delay on infinite horizon. From another point of view, the adjoint equation in our paper is given by a mean-field ASDE, which is totally different from that given by a forward-backward stochastic delay differential equation in [3].

Our paper is organized as follows. In Section 2, we give some preliminary results for the MF-BSDDE. The existence and uniqueness of solution of the mean-field BSDE with average delay on infinite horizon is proved in Section 3. Section 4 is devoted to the necessary condition for an infinite horizon optimal control problem with partial information. In Section 5, we formulate the corresponding sufficient condition. In Section 6, we apply the result obtained in Section 5 to study linear-quadratic control problem on infinite horizon and give the optimal control in the form of the solution of the mean-field forward-backward stochastic differential filtering equation. Finally, Section 7 is a concluding remark.

2. Preliminary results

Let $W(t)$, $\bar{W}(t)$, $t\ge 0$ be two independent standard one-dimensional Brownian motions, on a filtered probability space, $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\ge0},P)$ satisfying the usual conditions. Denote by $E[\cdot]$ the expectation under P. To simplify our notation, we will denote by $\tilde{\varrho}:=E[\varrho]$, $\tilde{\varrho}(t)=E[\varrho(t)]$, for any random variable ϱ or random process $\varrho(\cdot)$ unless otherwise stated.

Let us first give the following mean-field BSDE with average delay and propose some assumptions on the coefficient f, which are essential for our purpose.

$$\begin{cases} dY(t) = -f(t,\mathbf{Y}(t),Z(t),\bar{Z}(t),E[\mathbf{Y}(t)],E[Z(t)],E[\bar{Z}(t)])dt \\ \qquad\qquad +Z(t)dW(t)+\bar{Z}(t)d\bar{W}(t), \quad 0\le t\le\tau, \\ \lim\limits_{t\to\tau}Y(t)=\xi(\tau)\mathbf{I}_{[0,\infty)}(\tau), \quad Y(t)=\psi(t), \quad t\in[-\delta,0], \end{cases} \tag{2.1}$$

where $\tau\le\infty$ is a given $\mathscr{F}_t$-stopping time, possibly infinite, $\mathbf{Y}(t)=(Y(t),Y_2(t))$, with $Y_2(t)=\int_{t-\delta}^{t}e^{-\rho(t-r)}Y(r)dr$, and $\psi(t)$ is a given continuous function on $[-\delta,0]$. Throughout this paper, we introduce the following notations

$\delta>0$, $\rho>0$ are given constants,
f: $[0,\infty)\times R^2\times R\times R\times R^2\times R\times R\to R$.

Now we make the following assumptions:

($H1$) The function $f(\cdot,y,y_2,z,\bar{z},\tilde{y},\tilde{y}_2,\tilde{z},\tilde{\bar{z}})$ is progressively measurable for all y, y_2, z, $\bar{z}$, $\tilde{y}$, $\tilde{y}_2$, $\tilde{z}$, $\tilde{\bar{z}}$, and there exists a constant $C>0$, such that

$$\begin{aligned} &|f(t,y,y_2,z,\bar{z},\tilde{y},\tilde{y}_2,\tilde{z},\tilde{\bar{z}})-f(t,y',y_2',z',\bar{z}',\tilde{y}',\tilde{y}_2',\tilde{z}',\tilde{\bar{z}}')| \\ &\quad \le C(|y-y'|+|y_2-y_2'|+|z-z'|+|\bar{z}-\bar{z}'| \\ &\qquad +|\tilde{y}-\tilde{y}'|+|\tilde{y}_2-\tilde{y}_2'|+|\tilde{z}-\tilde{z}'|+|\tilde{\bar{z}}-\tilde{\bar{z}}'|). \end{aligned}$$

($H2$) There are real numbers β, C for sufficiently small ϵ, we have that

$$\beta>\left(\frac{1}{2}+2C+\frac{7C^2}{\epsilon}\right).$$

($H3$) We have a final condition on ξ, which is $\mathscr{F}_\tau$-measurable such that $E[e^{\beta\tau}|\xi|^2]<\infty$ and

$$E\left[\int_0^\infty e^{\beta t}|f(t,0,0,0,0,0,0,0,0)|^2dt\right]<\infty.$$