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Line-of-sight based spacecraft attitude and position tracking control

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ABSTRACT

This paper considers the problem of formation control of three spacecraft consisting of one leader and two followers. The leader spacecraft controls its attitude and position to track a desired attitude and position trajectory in the Earth Centred Inertial (ECI) frame. Each follower spacecraft tracks a desired relative attitude and relative position trajectory with respect to the leader spacecraft. Absolute attitude control law for the leader and relative attitude control laws for the followers are obtained in terms of line-of-sight vectors between the spacecraft. A relative attitude determination scheme using line-of-sight vectors is also proposed. The state feedback laws proposed in this work guarantee almost global asymptotic stability of the desired closed-loop equilibrium.

1. Introduction

Spacecraft formation flying has been extensively studied in recent years. The idea that a group of spacecraft flying as a formation can act as one large virtual instrument that is more efficient, more robust and less costly than a large monolithic spacecraft of similar capabilities has significant appeal. Simbol-X [8], MAXIM [9], NEAT [17] are among several spacecraft formation flying interferometry missions that were planned in the last decade. For a spacecraft formation to work as a virtual instrument, it is essential that the spacecraft in formation maintain precise relative attitudes and positions with respect to each other. From the control theory perspective, this poses a combined attitude and position control problem.

There have been several articles dealing with precise relative position control of spacecraft in formation, primarily utilizing navigation data from GPS [25]. The set of all possible attitudes of a rigid body is the set of 3×3 real orthogonal matrices with determinant 1, commonly referred to as the special orthogonal group SO(3). The special orthogonal group SO(3) is not a Euclidean space. However classical approaches to attitude control make use of local representations like Euler angles and non-unique representations like quaternions. It was only recently that rigid body attitude control problems were analysed in SO(3). Some examples of rigid body attitude control in SO(3) are given in [19,7]. In this paper, the attitude control system is constructed in SO(3), thus avoiding singularities associated with local parametrizations and ambiguity of quaternions.

Most of the existing relative attitude control laws (for example [20]) follow a common framework. It is assumed that absolute attitude of each spacecraft is measured independently with respect to a common inertial frame and are communicated to each other so as to calculate relative attitude. As relative attitudes are calculated indirectly, the accuracy is limited by attitude sensors of either spacecraft. Line-of-sight (LOS) vectors are a convenient way to measure relative attitudes. Line-of-sight observations between spacecraft can be found by standard light-beam focal-plane detector technology or by laser communication hardware as given in [11]. Cyclic formation control of spacecraft using inertial line-of-sight was proposed in [12]. The notion of attitude determination of a spacecraft from three independent vector observations in the spacecraft's own frame was proposed in [23], where the authors gave TRIAD and QUEST algorithms for attitude determination. More recently, authors in [16] suggest a scheme for determining relative attitude between two spacecraft using line-of-sight measurements to each other and to a reference body. Authors in [1] proposed a relative navigation scheme based on line-of-sight vectors. However their contribution did not include a control design. Both [2] and [24] propose attitude control laws for a single spacecraft making use of inertial line-of-sight measurements. Attitude stabilization of a single spacecraft with internal actuation using vector observations is solved in [2], while angular velocity free attitude control of single spacecraft using inertial line-of-sight measurements was proposed in [24]. Attitude synchronization in SO(n) of multiple agents sharing only a common link dependent vector is shown in [21]. Lee [15] proposes a control law to asymptotically stabilize relative attitude between two spacecraft

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making use of line-of-sight direction observations between them, and line-of-sight direction observations to a common object. In [15] control torques are obtained in terms of line-of-sight vectors. Authors in [30] extend the work of [15] to achieve tracking control of relative attitudes between multiple spacecraft. However in all these articles, spacecraft positions were assumed to be fixed during attitude manoeuvre. The Lyapunov analysis used in several earlier contributions like [24,15,30] could not allow translation since it would vary the line-of-sight vectors. Hence, the assumption that spacecraft positions are fixed was essential to prove closed loop stability in aforementioned results. This is restrictive as spacecraft formation applications require simultaneous control of attitude and position of the spacecraft. In this paper, we propose combined position and attitude control for a three spacecraft formation using line-of-sight measurements while assuming a serial communication architecture.

In [26,27], the authors proposed a novel line-of-sight based relative attitude control law that allowed translational motion of spacecraft during attitude manoeuvres. However control of position dynamics was not explicitly considered in [26]. This paper extends the concept to a three spacecraft formation with serial network architecture under gravity. The leader spacecraft controls its absolute position and absolute attitude with respect to an inertial frame so as to track a desired attitude and position trajectory. Two follower spacecraft control their relative position and relative attitude with respect to the leader, to track desired relative position and attitude trajectories. All attitude control laws are obtained in terms of line-of-sight unit vectors between spacecraft without using any external source. Leader spacecraft measures line-of-sight unit vectors to the follower spacecraft in its own body frame and communicates the same. The resulting closed loop system is shown to be almost globally asymptotically stable. A preliminary version of these results with a two spacecraft leader–follower architecture is given in [28].

In [31], simultaneous attitude and translational tracking control is considered assuming translational dynamics of the spacecraft to be double integrator. While the problem formulation is similar, this work provides a different control scheme and stability analysis. Compared to [31], the spacecraft translational dynamics is considered to be Keperian instead of double integrator and the work here makes no assumption about dynamics of line-of-sight unit vectors. In addition, this paper provides a novel framework to help reformulate several geometric attitude control schemes based on trace like error functions to line-of-sight control schemes. This last part is discussed further in Section 6.

Trace and modified trace attitude error functions are widely used in geometric control literature ([18,19,7] among many others). In this work, a trace like attitude error function is obtained in terms of line-of-sight vectors. Thus, the attitude control torques obtained by our proposed scheme are in the same spirit as aforementioned geometric control schemes. However, contrary to standard geometric control schemes using the trace error function, estimation of absolute or relative attitudes to implement the attitude control scheme is not required.

The paper is organized as follows, Section 2 mathematically formulates the problem, Section 3 describes the proposed attitude determination scheme, Section 4 describes error functions that are used later in the paper, Section 5 develops the control law and provides stability results for the closed loop system. Discussion of the results are provided in Section 6, numerical simulations are given in Section 7, and the conclusions are summarized in Section 8.

2. Problem formulation

2.1. Mathematical preliminaries

Spacecraft attitude dynamics are modelled as rigid body dynamics. Spacecraft attitude and translation dynamics are assumed to be decoupled. The rigid body attitude evolves over SO(3), a compact manifold given by

$$\mathrm{SO}(3)=\{R\in\mathbb{R}^{3\times 3}\,|\,R^\top R=RR^\top=I,\det(R)=1\}. \tag{1}$$

Because of the topological properties of SO(3), no globally asymptotically stable equilibrium exists under continuous control [3]. SO(3) forms a Lie group under the matrix multiplication operation. The lie algebra (tangent space at identity) of SO(3) is denoted as so(3) [13] and defined as,

$$\mathrm{so}(3)=\{S\in\mathbb{R}^{3\times 3}\,|\,S=-S^\top\}. \tag{2}$$

The map $\wedge:\mathbb{R}^3\to so(3)$, often called the “hat map”, denotes the isomorphism from $\mathbb{R}^3$ to $so(3)$. If $x=[x_1,x_2,x_3]^T$, $x\in\mathbb{R}^3$

$$\hat{x}=\begin{bmatrix}0 & -x_3 & x_2\\ x_3 & 0 & -x_1\\ -x_2 & x_1 & 0\end{bmatrix}. \tag{3}$$

Further $\hat{x}$ is a skew symmetric matrix satisfying, $\hat{x}y=x\times y$, $\forall y$. The inverse of the hat map is denoted as $(\cdot)^\vee:\mathrm{so}(3)\to\mathbb{R}^3$ and can be implicitly defined as $(\hat{x})^\vee=x$. The map $\mathrm{skew}:\mathbb{R}^{3\times 3}\to\mathrm{so}(3)$ is defined as $\mathrm{skew}(A):=\frac{A-A^\top}{2}$. It is evident from above that for all $A\in\mathbb{R}^{3\times 3}$, $(\mathrm{skew}(A))^\vee$ is well defined. Also, let $\mathrm{sym}(A):=\frac{A+A^\top}{2}$ and notice that $A=\mathrm{sym}(A)+\mathrm{skew}(A)$. Let tr() be the trace of a square matrix, defined as the sum of its diagonal elements.

2.2. Dynamics

Equations of motion of the i-th (for $i=1,2,3$) spacecraft orbiting earth under gravity, are given by

$$\dot{r}_i=v_i \tag{4}$$

$$\dot{v}_i=-\frac{\mu r_i}{\|r_i\|^3}+u_i \tag{5}$$

where $r_i\in\mathbb{R}^3$ is the position vector and $v_i\in\mathbb{R}^3$ is the velocity vector of the centre of mass of spacecraft i in ECI frame, $u_i\in\mathbb{R}^3$ is the force applied per unit mass and $\mu=3.98658366\times 10^{14}\ \mathrm{m}^3\ \mathrm{s}^{-2}$ is the gravitational parameter of Earth.

Let the attitude of spacecraft i be given by the rotation matrix R_i representing the transformation from body fixed frame of spacecraft i to a common inertial frame. Attitude dynamics of the i-th spacecraft, for $i=1,2,3$ are given by

$$\dot{R}_i=R_i\hat{\Omega}_i \tag{6}$$

$$J_i\dot{\Omega}_i=J_i\Omega_i\times\Omega_i+\tau_i \tag{7}$$

where $J_i\in\mathbb{R}^{3\times 3}$ is the moment of inertia, $\Omega_i\in\mathbb{R}^3$ is the angular velocity, and $\tau_i\in\mathbb{R}^3$ is the control torque of i-th spacecraft. Ω_i and τ_i are represented in the i-th spacecraft's body fixed frame.

2.3. Formation specifications

Consider a spacecraft as illustrated in Fig. 1. (x,y,z) is the Earth Centred Inertial reference frame. Axes (X',Y',Z'), (X'',Y'',Z'') and (X''',Y''',Z''') are the body fixed frames of spacecraft one, two and three, respectively.

Here spacecraft 1 is the leader while spacecraft 2 and 3 are the followers. It is required that the leader spacecraft should track a desired absolute attitude and position trajectory. Let $r_1^d(t)$ and $v_1^d(t)$ be the desired position and velocity of the leader spacecraft