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Symplectic method based on generating function for receding horizon control of linear time-varying systems

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ABSTRACT

A novel method for solving the linear receding-horizon control (RHC) problem with time-varying coefficients is proposed based on a generating function and the standard symplectic form of Hamiltonian systems. In contrast to other methods used to solve the linear RHC problem, the generating function is utilized to avoid directly online integrating the differential Riccati equation (DRE). Solutions to the DRE at discrete time points have been obtained by applying the generating function at each computation step. The derivation of the coefficient includes calculating the state transition matrices of the linear Hamiltonian system using the Magnus method, which preserves the symplectic structure of the Hamiltonian system. Numerical simulations of spacecraft rendezvous demonstrate that the proposed symplectic method obtains highly precise results for relatively long discretization sizes, and then yields computational efficiency improvements of one to two orders of magnitude compared with conventional backward sweep methods and the Legendre pseudospectral methods.

1. Introduction

The linear receding-horizon control (RHC) problem with time-varying coefficients plays a fundamental role in control engineering [1–6]. In recent years, there has been continued research in practical engineering with regard to the problems of real-time computation and the online implementation of RHC [7–17].

RHC problems can be categorized as two groups according whether constraints are considered or not. RHC with constraints has been successfully applied in the field of chemical engineering [7], industrial automation [8,9], et al. A characteristic feature of these engineering problems is that many complex constraints including state constraints and control constraints should be satisfied in the control process. As is well known, RHC can deal effectively with these constrained problems. Equally important, RHC without constraints has successfully solved the control problems originated from mechanical engineering [10–13] and aerospace engineering [14–17] in recent years. For high-speed mechanical engineering, a principal demand on any machining process is that operating time should be as short as possible, and notably chatter vibrations due to the regenerative effect limit the achievable results. Because the high-speed chatter vibrations are extremely small and hydraulic actuator can provide force/moment as expected, RHC without constraints can prevent chatter vibrations effectively [10,11]. For aerospace guidance engineering, the optimal reference trajectory of spacecraft/vehicle has been designed and calculated by off-line open-loop optimal control, and the real trajectory of spacecraft/vehicle is around the reference trajectory. Due to the environmental interference, navigation errors and model errors et al., the real trajectory will have a small deviation from the optimal reference trajectory and should be corrected. Because the deviation is small and needs no large control force/moment, RHC without constraints can correct deviation effectively [14,15]. Therefore, RHC without constraints have been widely used in high-speed mechanical engineering and aerospace guidance engineering. However, when the controlled linear system is a high-speed time-varying system, the sample period should be extreme small, i.e., the sample frequency is extreme high. Thus, the online computational efficiency is a crucial factor for successful implementation of unconstrained RHC.

Numerical methods with high efficiency for unconstrained RHC problems have attracted much attention and vigorous researches have been performing [15–24]. Owing to the exponential convergence, pseudospectral approximation methods

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including Legendre pseudospectral methods [16,17] and Radau pseudospectral methods [15] have been applied to RHC problems without constraints. In backward-sweep methods [18], the stabilizing feedback gains are obtained by integrating a differential Riccati equation (DRE) backward in time over a finite interval. In Ref. [19], it is shown that the unconstrained receding-horizon control problem can be converted to an initial-value problem for an ordinary differential equation. A simple model predictive control is obtained for solving the optimality conditions by a fixed-point iteration scheme in reference [20]. An approximate solution of linear time-varying RHC is obtained by first and higher order difference methods in reference [21]. By exploring the properties of the Riccati difference equation, a model predictive control for linear-time invariant systems without constraints is developed in reference [22]. A new stabilizing RHC scheme is proposed for linear discrete time-varying systems by linear matrix inequality [23]. The method proposed by Kowalska [24] employs variable step sizes to solve RHC problem.

From the above references, one of the most important problems in implementing unconstrained RHC law is the online computation burden associated with solving the DRE. Traditional methods, such as the Runge-Kutta method [25,26], are reported in Refs. [27–29]. These numerical methods should be implemented online for RHC. The online computational burden when solving the DRE poses a substantial obstacle to real-world deployment. Therefore, Lu [21] has given the approximate control law without solving the DRE. However, for some complex linear parameter-varying systems, we must employ high-order control laws with small time intervals to obtain highly accurate results. Unfortunately, higher order controllers of complex linear parameter-varying systems cannot be conveniently obtained for practical applications.

In contrast to the Taylor expansion employed in Ref. [21], this paper proposes an algorithm with high-performance based on the generating function method for linearly unconstrained RHC problems with time-varying coefficients. Indeed, the generating function method has been utilized to obtain the minimum H_∞ norm of linear-invariant systems [30] and linear-invariant systems with terminal constraint conditions. Inspired by reference [30], we have found a new method of avoiding the online integration of the DRE and have revealed that the results of the DRE can be obtained using the generating function. The resulting method is equivalent to the backward sweep method but without incorporating the online integration process. Therefore, the proposed method guarantees the stability of closed-loop systems. In addition, in the computational process of the present method, the solution to the DRE has been obtained via the coefficient operation of the generating function. This coefficient operation produces a standard symplectic matrix pair; thus, the symplectic structure of the solution to the DRE is preserved. In deriving the coefficient operation, state transition matrices (STMs) corresponding to the linear Hamiltonian system with time-varying coefficients are computed using the Magnus method, which preserves the symplectic structure of the linearly controlled Hamiltonian system [31–33]. Consequently, the standard symplectic matrix pair and the Magnus method provide more reliable solutions for the optimal trajectory and feedback control laws [30,34].

The advantages of the proposed method are the online computation burden has been substantially decreased, and a large time interval can be used while obtaining highly accurate results. Thus, the solution to the DRE has been obtained using the generating function without online solving of the DRE. Besides, because a highly accurate solution to the DRE has been obtained using the proposed numerical algorithm, a large time step can be utilized in practical applications.

2. Receding horizon control with time-varying coefficients

The linear RHC problem with time-varying coefficients and some relevant results are reviewed in this section. The linear differential equation of a controlled system with time-varying coefficients is expressed as follows

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)\mathbf{u},\ \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

where $\mathbf{x} \in R^{n\times 1}$ is the state, $\mathbf{u} \in R^{m\times 1}$ is the control input, $\mathbf{A}(t) \in R^{n\times n}$and $\mathbf{B}(t) \in R^{n\times m}$ are the time-varying coefficient matrices, and $\mathbf{x}_0$ is the initial condition. The system (1) in this paper is assumed to be uniformly completely controllable.

The linear RHC problem with time-varying coefficients at any fixed time $t \ge 0$ is defined as a linear optimal control in which the cost

$$J = \frac{1}{2}\int_t^{t+T} \left[\mathbf{x}^{\mathrm{T}}(\tau)\mathbf{Q}(\tau)\mathbf{x}(\tau) + \mathbf{u}^{\mathrm{T}}(\tau)\mathbf{R}(\tau)\mathbf{u}(\tau)\right] d\tau \tag{2}$$

is minimized for some chosen $\delta \le T < \infty$ (δ is a positive value) subject to the controlled system (1) with the terminal constraint condition

$$\mathbf{x}(t+T) = 0 \tag{3}$$

where $\mathbf{Q}(\tau) \in R^{n\times n}$ and $\mathbf{R}(\tau) \in R^{m\times m}$ are time-varying weighted matrices. The weighted matrix $\mathbf{Q}(\tau)$ is a positive semi-definite, symmetric matrix, and $\mathbf{R}(\tau)$ is a positive definite symmetric matrix.

By introducing a costate variable $\boldsymbol{\lambda}$, the optimal control input is obtained via the calculation of variations as follows:

$$\mathbf{u}(t) = -\mathbf{R}^{-1}(t)\mathbf{B}^{\mathrm{T}}(t)\boldsymbol{\lambda}(t) \tag{4}$$

Additionally, the optimal solution corresponding to Eq. (1) and the cost defined by Eq. (2) is obtained as follows:

$$\begin{Bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\boldsymbol{\lambda}}(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{A}(t) & -\mathbf{B}(t)\mathbf{R}^{-1}(t)\mathbf{B}^{\mathrm{T}}(t) \\ -\mathbf{Q}(t) & -\mathbf{A}^{\mathrm{T}}(t) \end{bmatrix} \begin{Bmatrix} \mathbf{x}(t) \\ \boldsymbol{\lambda}(t) \end{Bmatrix} \tag{5}$$

The solution of the linear Hamiltonian canonical Eq. (5) with time-varying coefficients can be expressed using the STM. The present states and costates at time t are expressed by the initial states and costate, i.e.,

$$\begin{Bmatrix} \mathbf{x}(t) \\ \boldsymbol{\lambda}(t) \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{xx}(t;t_0) & \boldsymbol{\Phi}_{x\lambda}(t;t_0) \\ \boldsymbol{\Phi}_{\lambda x}(t;t_0) & \boldsymbol{\Phi}_{\lambda\lambda}(t;t_0) \end{bmatrix} \begin{Bmatrix} \mathbf{x}(t_0) \\ \boldsymbol{\lambda}(t_0) \end{Bmatrix} \tag{6}$$

The STM $\boldsymbol{\Phi}(t;t_0) = \begin{bmatrix} \boldsymbol{\Phi}_{xx}(t;t_0) & \boldsymbol{\Phi}_{x\lambda}(t;t_0) \\ \boldsymbol{\Phi}_{\lambda x}(t;t_0) & \boldsymbol{\Phi}_{\lambda\lambda}(t;t_0) \end{bmatrix}$ is proven to be a symplectic matrix, i.e., the STM satisfies the following definition of a symplectic matrix:

$$\boldsymbol{\Phi}^{\mathrm{T}}(t;t_0)\mathbf{J}\boldsymbol{\Phi}(t;t_0) = \mathbf{J} \tag{7}$$

where $\mathbf{J}$ is a unitary symplectic matrix, i.e., $\mathbf{J} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{bmatrix}$

To define the relationship between the state $\mathbf{x}(t)$ and costate $\boldsymbol{\lambda}(t)$, we substitute Eq. (3) into the first row of Eq. (6) at time $t_0 = t$ and $t = t+T$ and then obtain

$$\mathbf{x}(t) = -\boldsymbol{\Phi}_{xx}^{-1}(t+T;t)\boldsymbol{\Phi}_{x\lambda}(t+T;t)\boldsymbol{\lambda}(t) = \mathbf{P}(t,t+T)\boldsymbol{\lambda}(t) \tag{8}$$

If the length of the receding horizon interval T is not infinite, the inverse of the $\boldsymbol{\Phi}_{xx}(t+T;t)$ in Eq. (8) always exists [25].