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Robust stability and performance analysis based on integral quadratic constraints[☆]

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Abstract

The integral quadratic constraints (IQC) approach facilitates a systematic and efficient analysis of robust stability and performance for uncertain dynamical systems based on linear matrix inequality (LMI) optimization. With the intention to make the IQC analysis tools more accessible to control scientists and engineers, we present in this paper a tutorial overview in three main parts: i) the general setup and the basic IQC theorem, ii) an extensive survey on the formulation and parametrization of multipliers based on LMI constraints, and iii) a detailed illustration of how the tools can be applied.

Keywords: robust stability and performance analysis, integral quadratic constraints, linear matrix inequalities

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1. Introduction

This paper is concerned with the analysis of uncertain dynamical systems based on the integral quadratic constraints (IQC) approach introduced by [86]. The origins of this approach can be dated back to the theories of absolute and input-output stability [26] as well as dissipative dynamical systems [133, 134], which were all developed during the sixties. The problem of absolute stability was concerned with the feedback interconnection of a linear time-invariant system with a static nonlinearity characterized by sector conditions, commonly referred to as the “Lur'e system”. The term “absolute” was coined since the analysis was concerned with an entire class of nonlinearities (rather than a particular one). Frequency domain absolute stability conditions like Popov and circle criteria became popular in the classical era as they were amenable to graphical checks in the way people knew from the Nyquist criterion. The breakthrough works of Yakubovich and Kalman established a link between the frequency domain condition and a linear matrix inequality (LMI), which was by then considered particularly useful for bringing the analysis into the frequency domain. Developed for a similar feedback configuration, the input-output stability framework offered the elegant small-gain and passivity theorems, which provided useful analysis results especially together with loop transformations. A breakthrough idea within the input-output stability theory was the introduction of so-called “multipliers” [135] within the loop in a way to obtain potentially less conservative stability conditions. Multipliers are artificial transfer functions that are required to satisfy certain conditions in connection with the considered uncertainty. Originally proposed by O'Shea [88, 89] and then formalized by Zames and Falb [136], the use of non-causal multipliers is marked as a major achievement of the input-output stability theory [17].

In the robust control era of the eighties, the focus shifted towards the analysis of multi-input multi-output, linear and time-invariant systems in the face of structured dynamic and parametric uncertainties that could even be mixed [30]. In this era, the term “robust” is coined to stability and performance when these properties are ensured for all uncertainties under consideration. Initial connections of robust stability analysis were already established to the multiplier theory (in particular to the circle criterion) early on in the eighties [107]. With linear systems in the interconnection, the formulation of the robust stability analysis problem then evolved into a complex structured singular value (commonly referred to as μ) analysis problem over the whole frequency axis. This problem is then rendered tractable by restricting the search to a (sufficiently dense) frequency grid and to the computation of upper bounds for μ, in which the so-called “scaling matrices” (D, DG) took the role of the multipliers [90]. Starting from the mid-phase of the robust control era, significant progress within the theory of optimization led to the development of efficient computational tools for control system analysis and synthesis based on LMI optimization. The IQC theorem of [86] unified all the classical results with those of robust control in the midst of this phase. As an efficient search of the IQC multipliers has been facilitated by LMI optimization, it has thus become possible to analyze robust stability against a wide variety of uncertainty classes and also to investigate system performance based on measures that can be

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