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Optimality-based dynamic allocation with nonlinear first-order redundant actuators

Thomas E. Passenbrunner [a], Mario Sassano [b,*], Luca Zaccarian [c,d]

[a] Institute for Design and Control of Mechatronical Systems, Johannes Kepler University Linz, 4040 Linz, Austria
[b] Dipartimento di Ingegneria Civile e Ingegneria Informatica, "Tor Vergata", Via del Politecnico 1, 00133 Rome, Italy
[c] CNRS, LAAS, 7 avenue du colonel Roche, F-31400 Toulouse, Univ. de Toulouse, LAAS, F-31400 Toulouse, France
[d] Dipartimento di Ingegneria Industriale, University of Trento, Italy

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ABSTRACT

A scheme is proposed to induce optimal input allocation of dynamically redundant nonlinear actuators with first-order dynamics that satisfies suitable regularity and stability assumptions. The allocation scheme is parametrized by a cost function associated with the most desirable actuator configuration, and guarantees convergence to the desired set point as well as to the minimum of the cost function. The overall scheme is also shown to reduce, in some special cases, to a nonlinear version of a PI type of control action.

1. Introduction

Redundant actuators typically characterize situations where the number of actuators available for control purposes is larger than the number of plant outputs to be regulated. This redundancy must be tackled by allocating the redundant inputs according to a given optimality criterion, which may be accomplished by way of a static or a dynamic control allocation scheme [10]. The optimality criterion may be motivated by the specific application and it may be characterized in terms of minimization of desired cost functions, such as energy consumption, risk of failure or safety considerations, to mention just a few.

Control allocation techniques arise from the legacy of mostly application oriented solutions (e.g., in the aerospace [12] and underwater [6] fields). They have been a topic of intense theoretical research activity in recent years, leading to several important schemes, well surveyed in [10,11] and, among others, in [5,19] and references therein. Most existing allocation techniques address the problem of linear actuators and correspond to static solutions minimizing some cost function at each time instant (see, e.g., [11]), which is often captured by intuitive type of goals such as mid-ranging (see [8] and references therein).

Perhaps the first paper using allocator dynamics is [9], where a gradient-based law is proposed in the presence of static actuators and nonlinear costs to be optimized. Later, dynamics have been used in allocators only in [20,18] and their applications [18,4,3]. Follow-up derivations related to the linear case can be found in [7,16] where the allocation problem is cast using regulation theory. Besides these works, as easily understandable from the comments in [10, Section 2.2.6], not much work has been done within the dynamic input allocation context, to date.

In this paper we tackle nonlinear actuators and unlike [9] where static nonlinear actuators are considered, we consider actuator dynamics described by strictly proper nonlinear differential equations satisfying some mild regularity conditions which appear to be quite reasonable for a set of actuators (see Assumption 1 and Remark 2 for a detailed discussion of such conditions). We propose a static state feedback allocation law which corresponds to an intermediate step before the introduction of our main contribution, namely a dynamic output feedback scheme that solves a *setpoint regulation* problem, minimizing at the same time a desired optimality criterion. The considered class of actuators encompasses, for instance, dynamically redundant actuators the dynamics of which have been identified using Wiener-type models, as explained in greater detail in the numerical example of Section 5.

A preliminary version of this paper appeared in [14]. As compared to that preliminary work, we provide here a more accurate problem definition, which allows to streamline the statement of

* Corresponding author.
E-mail addresses: thomas.passenbrunner@jku.at (T.E. Passenbrunner), mario.sassano@uniroma2.it (M. Sassano), zaccarian@laas.fr (L. Zaccarian).

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the main theorems and to avoid additional conditions required in [14], and we give the proofs of our main results, which were missing in [14]. In addition, numerical simulations are carried out here in the presence of unmodeled dynamics, demonstrating the intrinsic robustness of the approach. The paper is organized as follows. The considered allocation problem is introduced in Section 2. Section 3 provides the solution – in terms of a static state feedback – to the problem in the case of full information whereas the extension to a dynamic output feedback, i.e. when the actuator state is not available for feedback, is presented in Section 4. An application-motivated example is used to illustrate the performance of the proposed control law in Section 5. Finally, Section 6 contains all the technical derivations and the proofs of our main theorems.

2. Problem statement

As discussed in detail in, e.g., [5,10], the control architecture for over-actuated systems typically comprises three layers: a high-level motion control algorithm, a control allocation algorithm and a low-level control algorithm. These control layers, together with the plant, are interconnected according to a nested structure, possibly with the state of the plant being fed back to the high-level control algorithm. In this paper we focus on the control allocation task. As customary in input allocation problems, *virtual controls* $\tau \in \mathbb{R}^{n_\tau}$, see [10], should be suitably assigned by an allocator governing n_a actuators, with $n_a > n_\tau$, which comprises redundancy, to reproduce *commanded virtual controls* $\tau_c \in \mathbb{R}^{n_\tau}$ requested by the high level controller. In this paper we consider then a pool of n_a actuators obeying a first-order (possibly coupled) dynamics:

$$\dot{x}_a = f(x_a) + g(x_a)u, \quad \tau = h(x_a), \tag{1}$$

where $f(\cdot) : \mathbb{R}^{n_a} \to \mathbb{R}^{n_a}$, $g(\cdot) : \mathbb{R}^{n_a} \to \mathbb{R}^{n_a \times n_a}$ and $h(\cdot) : \mathbb{R}^{n_a} \to \mathbb{R}^{n_\tau}$.

Remark 1. While more general settings with actuators having higher-order state realizations are feasible, in this work we focus on the setting (1), which is already challenging due to the nonlinear coupling established by $h(\cdot)$. In practical cases, dynamics (1) often arise when data-driven identified actuator models with prescribed nonlinear structure are used in the allocation design, possibly with unmodeled dynamics. This is for example the case for the experiment described in Section 5.

Within our scheme, some robustness to unmodeled dynamics (such as, e.g., the fast electrical time constant of the actuators in Section 5) follows from intrinsic robustness of asymptotic stability under mild regularity conditions on the data of the control system (see, e.g., [17]). We make the following assumption on dynamics (1).

Assumption 1. The following holds:

(1) the functions $f(\cdot)$ and $g(\cdot)$ are locally Lipschitz and $h(\cdot)$ is continuously differentiable;
(2) the function $g(\cdot)$ is uniformly bounded from below, namely there exists a positive scalar g_m such that $g_m \le \min_i \sigma_i(g(x_a))$ for all $x_a \in \mathbb{R}^{n_a}$, where $\sigma_i(g)$ denotes the i-th singular value of g;
(3) the gradient of h, $\nabla h(x_a) \in \mathbb{R}^{n_a \times n_\tau}$ is full column rank for all $x_a \in \mathbb{R}^{n_a}$, namely, the matrix $(\nabla h(x_a))^T \nabla h(x_a)$ is nonsingular everywhere.

Remark 2. Assumption 1 is very mild if one keeps in mind that system (1) corresponds to actuators dynamics. Intuitively, they convey the fact that actuator dynamics should be characterized by sufficiently regular (differentiable) functions and that no controllability loss should be possible during any operation range of the actuators. More specifically, item 1 conveys mild regularity assumptions to ensure existence and uniqueness of solutions and for the gradient of item 3 to be well defined. Item 2 resembles the fact that the external input u of the actuating system affects the actuator dynamics in a consistent way throughout the whole operating range. Finally, item 3 corresponds to the requirement that in any operating condition of the actuators, each virtual control in τ can be effectively changed by a variation of at least one of the actuators' states x_a.

In this paper we address the problem of designing a dynamic allocator for actuators (1), which operates in feedback from the *virtual control* τ with the goal of guaranteeing suitable regulation of a *commanded virtual control* $\tau_c \in \mathbb{R}^{n_\tau}$, while ensuring that, at the steady-state, the actuators state x_a minimizes a desired (possibly nonlinear) cost function, subject to the constraint that the (higher priority) virtual control assignment task is accomplished. While assuming availability of τ may not be reasonable for some applications, alternative feedback schemes from suitable estimates of τ could be envisioned. We believe that these schemes would clearly emulate the output feedback solution proposed here and not add much challenges to the underlying theory. Therefore, due to lack of space, they are not pursued here where the attention is focused primarily on dynamics (1). This issue may be, for instance, circumvented by augmenting the dynamics (1) with an observer that reconstructs τ from accessible information, provided the extended system satisfies Assumption 1. The above goals are formalized next.

Goal 1. *Given actuators (1), a smooth cost function $x_a \mapsto J(x_a)$, and a regulation performance scalar parameter $\gamma_p > 0$, design a controller in feedback from τ, such that for each commanded virtual control τ_c and some non-empty set Ω,*

(i) stability: *a suitable subset of the manifold where $h(x_a) = \tau_c$ is uniformly asymptotically stable*;
(ii) setpoint regulation: *the closed-loop guarantees that*:
   1. *the commanded virtual control is asymptotically tracked from any initial condition in Ω, i.e. $\lim_{t\to\infty} |\tau(t) - \tau_c| = 0$;*
   2. *from a suitable initial condition of the controller, $|\tau(t) - \tau_c| = \exp(-\gamma_p t)|\tau(0) - \tau_c|$;*
(iii) asymptotic optimality: *for each τ_c such that function $x_a \mapsto J(x_a)$ restricted to the level set $h(x_a) = \tau_c$ is strictly convex, we have $\lim_{t\to\infty} x_a(t) = \bar{x}_a$, where:*

$$\bar{x}_a := \arg\min_{x_a \in \mathbb{R}^{n_a}} J(x_a), \quad \text{subject to } h(x_a) = \tau_c, \tag{2}$$

which is well-defined from strict convexity.

Moreover, if Ω coincides with the entire state-space, then the controller is said to globally solve the problem.

The high level control algorithm is assumed to be suitably designed, in the sense that it allows to achieve desired (asymptotic) stability properties, with some intrinsic robustness, within a control scheme similar to the one in Fig. 1 but in the absence of a control allocation algorithm. The output of the control allocation algorithm, namely the virtual control τ, which should reproduce the commanded virtual control τ_c, is the input to the low-level controllers of effectors and actuators in the plant. The commanded virtual input must be then interpreted as *set-point values* for the low-level actuators of the controlled plant, which are dictated by a desired steady-state reference for the plant. Moreover, item (ii.1) ensures that the DC gain of the virtual allocator is unitary. Instead the requirement in (ii.2) corresponds to ensuring that the high-level controller *sees* a virtual linear first-order decentralized dynamics whose speed can be arbitrarily assigned via parameter