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Adaptive remeshing based on a posteriori error estimation for forging simulation

Ramzy Boussetta, Thierry Coupez, Lionel Fourment *

CEMEF, École Nationale Supérieure des Mines de Paris, UMR CNRS 7635, BP 207, 06 904 Sophia Antipolis, France

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Abstract

This paper presents a fully automatic 3D adaptive remeshing procedure and its application to non-steady metal forming simulation. Remeshing, here, is considered as the improvement of an existing mesh rather than a complete rebuilding process. The mesh optimization technique is described. It is based on the combination of local improvement of the neighbourhood of nodes and edges. The surface and the volume remeshing are coupled by using a layer of virtual boundary elements. The mesh adaptation is performed by the optimization of the shape factor. The mesh size map enforcement is accounted for working in a locally transformed space. The size map is provided by a Zienkiewicz–Zhu type error estimator. Its accuracy is evaluated in the frame of a velocity/pressure formulation, viscoplastic constitutive equation and 3D linear tetrahedral elements, by numerical experiments. The adaptive remeshing procedure is applied to non-steady forging. Several complex 3D examples show the reliability of the proposed approach to automatically produce optimal meshes at a prescribed computational cost.

Keywords: Meshing; Adaptive remeshing; Error estimation; Forging

1. Introduction: Large deformation and adaptive remeshing

Unstructured mesh generation is a general solution for the construction of a mesh in complex geometries. In a certain number of domains as in forming process simulation, the industrial geometries are really complex. The meshing technique used in this work was developed to solve the remeshing stage in large deformations [1,2]. In such cases, the mesh deforms with the material domain (Lagrangian approach) and thus it degenerates rapidly. In fact, this meshing method is a complete solution in terms of adaptive meshing (including parallel remeshing [3]). Several examples of applications in material forming processes can be found in [4,5]. Although it is based on local improvement mechanism, it is really a mesh generation method which is clearly different from any other well established meshing tool: Delaunay, frontal or octree. The algorithm exploits the possibility to operate on the mesh topologies without considering any geometric constraint. A simple local process is used to derive from mesh to mesh. The node creation and deletion are implicitly contained in the method and the modification of the surface mesh and the volume mesh are strongly coupled. The technique is perfectly suited for forging simulation since it provides a way to repair (elements degeneracy due to the Lagrangian approach) as well as to adapt the mesh to the calculated solution.

A mesh can be adapted dynamically by deriving a new mesh from the old mesh with respect to a mesh size map. A posteriori estimation of the finite element discretization error allows computing an optimal map, which minimizes the mesh

* Corresponding author. Tel.: +33 4 93 95 75 95; fax: +33 4 92 38 97 52.
E-mail addresses: ramzy.boussetta@ensmp.fr (R. Boussetta), thierry.coupez@ensmp.fr, thierry.coupez@cemef.cma.fr (T. Coupez), lionel.fourment@ensmp.fr (L. Fourment).

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size for a prescribed accuracy. Many advances have been made in the development of reliable error estimators during the last years. They can be classified into three families: the residual type error estimators, firstly introduced by Babuška and Rheinboldt [6], the estimators based on the error in constitutive relation, developed by Ladevèze et al. [7] and the Zieniewicz–Zhu's (Z^2) [8] type error estimators. In this work, the Z^2 is followed as it is now widely used in many industrial codes due to its efficiency and reliability in estimating errors, as well as for its simplicity of implementation. It is based on the construction of a recovered stress tensor field, $\tilde{\sigma}_h$, more accurate than the finite element solution, σ_h. Initially, $\tilde{\sigma}_h$ was obtained by a variety of projection techniques, such as least square smoothing or simple nodal averaging of adjacent elements values [9]. The approach has been significantly improved by using the superconvergence properties of the finite element solution at some points, and by recovering an enhanced solution inside a local finite element patch. The so-called called superconvergent patch recovery (SPR) technique [10] have been used in a slightly different way by other authors [11,12] who preferred the Liszka–Orkisz local finite difference method [13] to compute the recovered solution from nodal [12] or Gauss integration points [11]. The present study utilizes the Z^2-SPR error estimator in its almost standard form.

The solvers based on tetrahedral meshes are now well established and more particularly finite element method works well both in the context of solid and fluid mechanics. We will focus on a 3D solver developed from a mixed finite element method based on the MINI element (P1+/P1). This solver can be used both in metal forming applications and polymer forming applications. That means that it is usable for small and large deformation of incompressible material and also for incompressible flow of hot polymer and finally to casting. This solver ranges from solid mechanics to fluid mechanics. Moreover, the use of a first order stable tetrahedra (the mini element), allows to have unstructured meshes and iterative solvers [14].

2. Mesh topology optimization

2.1. Mesh topology

The mesh optimization technique used in this paper is based on a simple local mechanism applying on mesh topologies. For that purpose let us introduce notations allowing to introduce precisely the mesh topology. A mesh is determined by a set of coordinates (the mesh node coordinates) and by a set of elements, each element being completely defined by the node numbers of its vertices. The mesh connectivity by means of the element node relations will be called the mesh topology. It can be described independently from any mesh node coordinate.

Let $N \subset \mathbb{N}$ be a finite set of nodes simply reduced to a set of numbers. Let $P_D(N)$ be the set of parts of N which are composed of exactly D different nodes. $\mathcal{T}$, a mesh topology, is a set of elements, $\mathcal{T} \subset P_D(N)$, having the properties described hereafter. The nodes of $\mathcal{T}$ are the subsets of nodes used by $\mathcal{T}$:

$$N(\mathcal{T}) = \bigcup_{T\in\mathcal{T}} T. \tag{1}$$

Let us consider only the mesh topologies associated with the simplex element (triangles, tetrahedra, etc.). A d-simplex is composed of $D = d + 1$ vertices. We can define ∂T the boundary of each element $T \in \mathcal{T}$ as the set of all subset of $D - 1$ nodes of T (the three edges of a triangle or the six faces of a tetrahedron). The set of faces of $\mathcal{T}$, $\mathcal{F}$, is then defined by

$$\mathcal{F}(\mathcal{T}) = \bigcup_{T\in\mathcal{T}} \partial T. \tag{2}$$

Finally, let us denote by

$$\mathcal{T}(\eta) = \{T \in \mathcal{T},\ \eta \subset T\} \tag{3}$$

the set of elements sharing a given subset of nodes η. We can introduce the following definition: $\mathcal{T}$ is a mesh topology if

$$\mathrm{card}(\mathcal{T}(F)) < 2, \quad \forall F \in \mathcal{F}(\mathcal{T}). \tag{4}$$

The boundary of $\mathcal{T}$ is

$$\partial\mathcal{T} = \{F \in \mathcal{F}(\mathcal{T}),\ \mathrm{card}(\mathcal{T}(F)) = 1\}. \tag{5}$$

2.2. Mesh topology operator

A local modification in a mesh is a cut and paste operation which is written as

$$\mathcal{T} \leftarrow \mathcal{T} - a + b, \tag{6}$$

where a is a the subset of elements in $\mathcal{T}$ to be replaced by b, another set of elements.