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Approximate solution for discharge coefficient of the sonic nozzle with surface roughness

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A R T I C L E I N F O

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A B S T R A C T

Sonic nozzle is widely used in the flow measurement and control. Nowadays, it has been applied to higher Reynolds number flow increasingly. The effect of surface roughness on discharge coefficient of the sonic nozzle should be discussed. An approximate analytic solution for discharge coefficient of the sonic nozzle with surface roughness was proposed in detail. The determination of this coefficient was based on universal logarithmic velocity-distribution law and the principle of equivalent velocity profile. Although there are some apparently approximations, this algebraic method accurately predicts the discharge coefficient of the sonic nozzle with surface roughness in Reynolds number range from 10^4 to 10^9, a relative equivalent roughness range of 10^{-6} to 10^{-2}. Some experiments of sonic nozzle conducted by others showed agreement with present algebraic method. Besides, the agreement between this method and the corresponding exact numerical calculation is also good. The present method provides an excellent tool to deeply investigate the roughness effect and promote further improvement of the standard.

1. Introduction

Sonic nozzle, with the advantages of high stability and accuracy, is widely applied to the flow measurement and control of the gas mass flow-rate [1]. The discharge coefficient C_d is the most important performance parameter for sonic nozzle. A large number of scholars, such as Hall [2], Stratford [3], Geropp [4], Arnberg [5], Ishibashi [6], Cruz-Maya [7] and Mickan [8] had obtained many theoretical formulas for calculating the discharge coefficient of sonic nozzle. In addition, there are many experiments were conducted to acquire the empirical formula of discharge coefficient [9–12]. But, most of these studies did not consider the effect of surface roughness on discharge coefficient.

At present, along with the wide use of sonic nozzle in high Reynolds number fields [13,14], the effect of surface roughness is getting more and more attention. Wendt [15] and Li [16] thought the roughness effect is relatively small at low or moderate Reynolds number. Gibson and Stewart [17] have reported a series of experiments to investigate the roughness effects of deviating from ISO 9300. It was found that there is a noticeable overall reduction in discharge coefficient. Krishnamurty [18] studied computationally the effect of roughness height on the performance of two-dimensional, converging-diverging nozzles. It is observed that the skin friction coefficient will decrease as the wall roughness increases which means discharge coefficient decreases. Anthony [19] investigated the roughness effect on the nozzle boundary layer transition. Alper [20] numerically evaluated C_d-value of choked converging nozzles with different relative roughness. It proposed that the influence of surface rough on C_d increases in the systems having lower mass flow rates. Wang [21,22] numerically investigated the roughness effect on C_d in a wide range of Reynolds number and relative roughness.

In this study, an approximate algebraic solution of nozzle flow with surface roughness, based on universal logarithmic velocity-distribution law and the principle of equivalent velocity profile, was presented. This algebraic method can well predicts C_d of sonic nozzle with surface roughness in a large Reynolds number range of 10^4 to 10^9 and relative equivalent roughness range from 10^{-6} to 10^{-2}.

2. Problem statement

The configuration of ISO 9300 toroidal-throat Venturi nozzle is shown in Fig. 1. The discharge coefficient C_d is the ratio of the actual mass flow-rate q_m to the ideal mass flow-rate q_{mi},

$$C_d = q_m/q_{mi} \tag{1}$$

The ideal critical mass flow-rate q_{mi} for one-dimensional isentropic flow of an ideal gas can be calculated by [23]

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Nomenclature

A	area, m^2
a	sonic speed, m/s
C^*	critical flow function, (dimensionless)
C_d	discharge coefficient, (dimensionless)
c_f	skin-friction coefficient, (dimensionless)
d	throat diameter of the nozzle, mm
F, G	defined in Eqs. (13) and (18).
K_s	equivalent sand roughness, m
k	thermal conductivity, W/(m K)
M	Mach number, (dimensionless)
Pr	Prandtl number, (dimensionless)
p	pressure, kPa
q_m	real gas mass flow-rate, kg/s
R_a	average roughness, m
R_c	curvature radius of nozzle wall, m
Re_d	Reynolds number based on throat diameter, (dimensionless)
Re_δ	Reynolds number based on δ, (dimensionless)
R_m	specific gas constant, J/(kg K)
T	temperature, K
u, v	Velocity components in x, y directions, m/s
u_τ	friction velocity, m/s
x, y, r	curvilinear system of coordinates

Greek

γ	isentropic exponent, (dimensionless)
δ	boundary-layer thickness, m
δ_1	displacement thickness, m
δ_2	momentum thickness, m
δ_3	energy thickness, m
θ	diffuser angle, °
κ	von Karman's constant, 0.4
λ	resistance coefficient, (dimensionless)
μ	dynamic viscosity, Pa s
υ	kinematic viscosity, m^2/s
Π	Coles' wake parameter, (dimensionless)
ρ	density, kg/m^3
τ_w	shearing stress at the wall, Pa
Ψ, ζ	defined in Eqs. (23) and (26).

Superscripts

+	denotes dimensionless parameters

Subscripts

0	at stagnation condition
1	main-stream condition
nt	at the nozzle throat
cr	critical point
w	wall

$$q_{mi} = \frac{A_{nt} C_i^* p_0}{\sqrt{R_m T_0}} \tag{2}$$

where, ideal gas critical flow function C_i^* is given by $\sqrt{\gamma(2/\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}$.

Three distinct types of theoretical models for discharge coefficient have been developed [24]. Firstly, viscous discharge coefficient C_{d1} is affected by gas viscosity (i.e. boundary layer), secondly, inviscid discharge coefficient C_{d2} is induced by multi-dimensional flow depending on geometry of nozzle [2], and lastly, virial discharge coefficient C_{d3} is caused by physical property of real gas. Using the accurate formulas from references [21,25], C_{d3} can be directly calculated and equal to the relative real gas critical flow factor C_r^*/C_i^*. Thus, in this study, the discharge coefficient just considers first two items.

According to Hall's equation, [2]

$$C_{d2} = 1 - \frac{\gamma+1}{(2R_c/d)^2}\left(\frac{1}{96} - \frac{8\gamma+21}{4608(2R_c/d)} + \frac{754\gamma^2+1971\gamma+2007}{552960(2R_c/d)^2}\right) \tag{3}$$

C_{d1} is attributed to the development of boundary layer along the nozzle wall. The predictive formulas (4) of C_{d1} have been developed both for laminar and turbulent flows [3,4,26]

$$C_{d1} = \begin{cases} 1 - \frac{4}{\sqrt{Re_d \cdot m}}\left(\frac{\gamma+1}{2}\right)^{\frac{1}{2(\gamma-1)}}\left(3\sqrt{2} - 2\sqrt{3} + \frac{\gamma-1}{\sqrt{3}}\right) & \text{, laminar flow} \\ 1 - 0.0525(d/R_c)^{-0.4} Re_d^{-0.2} & \text{, turbulent flow} \end{cases} \tag{4}$$

Where

$$Re_d = \frac{u_1 d}{\nu} = \frac{4q_m}{\pi d \mu_0} \tag{5}$$

and

$$m = \sqrt{\frac{2d}{R_c}\left(\frac{\gamma+1}{2}\right)^{\frac{3\gamma-1}{\gamma-1}}} \tag{6}$$

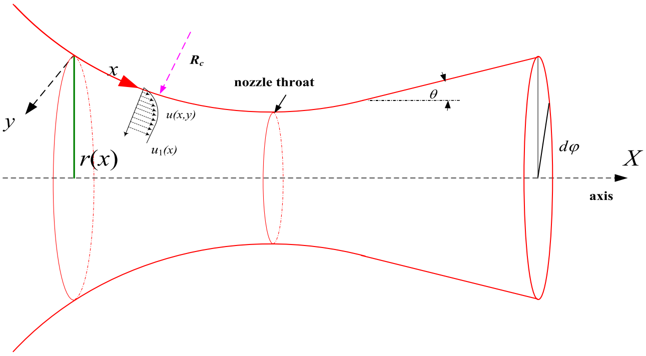

Fig. 1. ISO 9300 toroidal-throat Venturi nozzle.

All these models for turbulent flow are based on momentum or energy integral equations without the effect of the roughness on skin friction coefficient C_f. However, most nozzles cannot be regarded as being hydraulically smooth, at least at higher Reynolds numbers. The skin friction coefficient C_f of rough wall is larger than that of smooth wall [18]. Thus, the discharge coefficient will be affected by the wall roughness.

3. Approximate solution for discharge coefficient

3.1. Displacement thickness of the rough wall

Experimental data shows that the logarithmic law is valid in the turbulent region [27]. For the compressible flow with pressure gradients, it is satisfactory to correlate the whole velocity profile with a wall-wake law

$$u^+ = \frac{1}{\kappa}\ln y^+ + B' + \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right) \tag{7}$$