Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnline CONFERENCE PAPER ARCHIVE

IFAC-PapersOnLine 49-30 (2016) 132–137

Stability of Discrete-time Control Systems with Uniform and Logarithmic Quantizers

G. C. Campos[1], **J. M. Gomes da Silva Jr.**[1], **S. Tarbouriech**[2], **C. E. Pereira**[1]

1.Universidade Federal do Rio Grande do Sul (UFRGS), Dept. of Automation and Energy Syst. (DELAE), Porto Alegre, Brazil (e-mail: gustccam@gmail.com, jmgomes@ufrgs.br, cepereira@ufrgs.br)
2.LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, France (e-mail: tarbour@laas.fr)

Abstract: This paper deals with stability analysis of discrete-time linear systems involving finite quantizers on the input of the controlled plant. Two kinds of quantization are analyzed: uniform and logarithmic. Through LMI-based conditions, an attractor of the state trajectories and a set of admissible initial conditions are determined. A method is proposed to compare the performances of the two kinds of quantization in terms of the dimensions of the attractor, considering a scenario of Networked Control Systems (NCS). Computational issues are discussed and a numerical example is presented to validate the work.

Keywords: quantized control, stability, attractors, networked control

1. INTRODUCTION

The use of digital controllers became very frequent in control systems in the last decade. It is well known that in digital control, some undesired side effects such as time-delays, asynchronism, saturation and quantization are implicit. For instance, saturation may not only reduce the performance of a system, but also deteriorate stability. Similarly, the effect of quantization in control systems is a known phenomenon, which may lead to limit cycles, undesired equilibria or chaotic behavior, even if the controller is a stabilizing one (Delchamps [1990], Tarbouriech and Gouaisbaut [2012]). Now that Networked Control Systems (NCS) are becoming increasingly popular, researches on quantization regained attention of the control community. Since in NCS the control loop elements exchange information through communication channels with limited bandwidth, the control systems may become more susceptible to quantization side effects.

Then, this kind of communication constraints has attracted the attention of researchers over the last years; see, for example, Brockett and Liberzon [2000], Fridman and Dambrine [2009], Liberzon [2003]. It is also important to note that several studies have been proposed for continuous-time systems. In the context of discrete-time systems, one can cite several works dealing with the controller or observer design in presence of uniform or logarithmic quantizers: see, for example, Picasso and Colaneri [2008] and Xia et al. [2013]. In particular, in Elia and Mitter [2001], it has been shown that, for a quadratically stabilizable system, a logarithmic quantizer is the optimal solution in terms of coarse quantization density. However, it is also shown that the quantizer must have an infinite number of quantization levels, which is not possible to implement. Following the same idea, Fu and Xie [2005] have introduced the sector bound approach for quantized feedback systems giving simple formulae to the stabilization problem considering state and output feedback controllers. Many other works were carried out as in de Souza et al. [2010], where the sector bound approach is applied to derive LMI based conditions for estimating a set of initial conditions and an attractor, such that all the state trajectories starting in the first set will enter the attractor in a finite time and remain inside it. In that work the controller and a finite logarithmic quantizer are supposed to be given and the stability analysis problem is addressed.

Despite these results obtained for logarithmic quantizers, research on uniform quantizers is not less relevant. In Ferrante et al. [2015], sector conditions are used to find a compact invariant set surrounding the origin, which attracts all the state trajectories, considering two different settings involving uniform quantization in continuous-time linear systems. The controller is designed in order to minimize the dimensions of the attractor. Tarbouriech and Gouaisbaut [2012] also considers the effects of saturation to design a state feedback control law while minimizing the attractor and maximizing the set of admissible initial conditions.

This paper deals with the stability analysis of discrete-time linear systems involving saturation and either an uniform or a logarithmic finite quantizer. For simplicity, only the input quantization case is considered. Considering uniform quantizers, we extend the work of Ferrante et al. [2015] and Tarbouriech and Gouaisbaut [2012] to the discrete-time case. Using sector conditions and the S-procedure,

⋆ This work was in part supported by CAPES (projects PVE-129/2012 and STICAmSud) and CNPq (PQ 305979/2015-9 and 308605/2015-2), Brazil, and also by ANR project LIMICOS grant number 12BS0300501.

10.1016/j.ifacol.2016.11.142

uniform ultimate boundedness stability is analyzed for a given static state feedback controller, while the attractor and the set of admissible initial conditions are determined. Then an LMI-based optimization problem is proposed to minimize the attractor and simultaneously maximize the set of admissible initial conditions (which can be considered as an estimate of the region of attraction of the origin). For the logarithmic quantizers, a similar analysis is carried out. After the analysis of both uniform and logarithmic quantization, we propose a method to compare the performance of quantizers, in terms of attractor size, in a NCS scenario. Finally, some simulations are presented to validate the results.

Notation. Throughout the article, I denotes the identity matrix and 0 denotes the null matrix (equivalently the null vector) of appropriate dimensions. For a matrix $A \in \mathbb{R}^{n\times m}$, A', $A_{(i)}$, $tr(A)$ denote its transpose, its ith row and its trace respectively. The matrix $diag(A_1, A_2, \ldots, A_n)$ is the block-diagonal matrix having A_1, A_2, ..., A_n as diagonal blocks and in symmetric matrices $*$ stands for symmetric blocks. For a vector $x \in \mathbb{R}^n$, $x_{(i)}$, x', $|x|$ denote its ith component, its transpose and the componentwise absolute value operator respectively. $sign(x)$ is the componentwise sign function, with $sign(0) = 0$, and $\lfloor x \rfloor$ the componentwise floor operator. For two sets $\mathscr{S}_1$ and $\mathscr{S}_2$, $\mathscr{S}_1 \backslash \mathscr{S}_2$ denotes the set $\mathscr{S}_1$ deprived of $\mathscr{S}_2$.

2. STABILITY WITH UNIFORM QUANTIZERS

2.1 Problem statement

Consider the following discrete-time linear system:

$$\begin{cases} x(k+1) = Ax(k) + Bsat\Big(q\big(u(k)\big)\Big) \\ x(0) = x_0 \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $x_0 \in \mathbb{R}^n$ are respectively the state, the input of the system and the initial state. A, B are real matrices of suitable dimensions, and $q(\cdot)$ is a componentwise uniform quantizer, which is described by the static nonlinear function defined as

$$q_{(i)}(u) := \Delta sign(u_{(i)}) \left\lfloor \frac{|u_{(i)}|}{\Delta} \right\rfloor \quad (2)$$

where Δ is a positive real scalar representing the quantization step. The saturation map $sat(\cdot)$ represents a symmetric saturation function defined as follows:

$$sat(u_{(i)}) = sign(u_{(i)}) \min\{u_0, |u_{(i)}|\},\ i = 1, \ldots, p \quad (3)$$

with $u_0 > 0$ being the symmetric bounds on the control input $u_{(i)}, i = 1, \ldots, p$.

Assuming that the state x is fully accessible, we want to analyze stability of system (1) subject to the following control law $u = Kx$. Therefore, by defining the functions

$$\psi(v) := q(v) - v \quad (4)$$

$$\phi(v) := sat(v) - v \quad (5)$$

the closed-loop system becomes:

$$\begin{cases} x(k+1) = (A+BK)x(k) + B\psi(k) + B\phi(k) \\ x(0) = x_0 \end{cases} \quad (6)$$

with $\begin{cases} \psi(k) = \psi\big(u(k)\big) \\ \phi(k) = \phi\Big(q\big(u(k)\big)\Big) = \phi\Big(u(k) + \psi\big(u(k)\big)\Big) \end{cases}$

The presence of the uniform quantizer defined in (2) can represent a real obstacle to the asymptotic stabilization of the closed-loop system, due to its deadzone effect (Ferrante et al. [2015]). If the matrix A is not Schur-Cohn, the asymptotic stabilization of the origin for the closed-loop system cannot be achieved even when the gain K is supposed to be a stabilizing one (that is, when $(A + BK)$ is Schur-Cohn). Moreover, if A is not Schur-stable, under input saturation, only local (regional) stability can be achieved (Tarbouriech et al. [2011]). In this work, we focus on this case. Then the problem we aim to solve can be stated as follows.

Problem 1. Given the matrices A, B and a stabilizing gain K of adequate dimensions and a positive real quantization step Δ, determine a set $\mathscr{S}_0 \subset \mathbb{R}^n$ and a compact set $\mathscr{S}_u \subset \mathbb{R}^n$ containing the origin, such that

- $\mathscr{S}_0$ and $\mathscr{S}_u$ are invariant sets;
- For every initial condition $x_0 \in \mathscr{S}_0 \backslash \mathscr{S}_u$, the trajectories are bounded and converge in a finite time into $\mathscr{S}_u$ (which is therefore an attractor of the trajectories).

2.2 Main results

To solve Problem 1, we recall the sector conditions used in (Ferrante et al. [2015]) and (Tarbouriech and Gouaisbaut [2012]).

Lemma 1. For every $\psi(u)$ as defined in (4) with $u = Kx$, the following relations are verified:

$$\psi' S_1 \psi - tr(S_1)\Delta^2 \le 0 \quad (7)$$

$$\psi' S_2 (\psi + Kx) \le 0 \quad (8)$$

for any diagonal positive definite matrices S_1, $S_2 \in \mathbb{R}^{p\times p}$.

Lemma 2. For every $\phi(u)$ as defined in (5) with $u = Kx$ and every matrix $G \in \mathbb{R}^{p\times n}$, the following relation is verified:

$$\phi' S_3 \Big(sat\big(q(Kx)\big) + Gx \Big) \le 0 \quad (9)$$

for any diagonal positive matrix $S_3 \in \mathbb{R}^{p\times p}$, provided that $x \in S(u_0)$ with

$$S(u_0) = \big\{ x \in \mathbb{R}^n; -u_0 \le G_{(i)}x \le u_0, \forall i \in \{1, \ldots, p\} \big\} \quad (10)$$

Note that condition (9), combined with (4) and (5), reads:

$$\phi' S_3 (\phi + \psi + Kx + Gx) \le 0 \quad (11)$$

By using Lemmas 1 and 2, the following proposition to solve Problem 1 can be stated.

Proposition 1. If there exist a symmetric positive definite matrix $W \in \mathbb{R}^{n\times n}$, three diagonal positive matrices S_1, S_2, $R_3 \in \mathbb{R}^{p\times p}$, a matrix $Z \in \mathbb{R}^{p\times n}$, two positive scalars τ_1, τ_2 and a scalar α, $0 \le \alpha \le 1$, satisfying the conditions (12)–(14):

$$\begin{bmatrix} W(\tau_2 - \alpha\tau_1 - 1) & -WK'S_2 & -WK' - Z' & W(A+BK)' \\ * & -S_1 - 2S_2 & -I & B' \\ * & * & -2R_3 & R_3B' \\ * & * & * & -W \end{bmatrix} < 0 \quad (12)$$