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#### $\mathcal{H}_{\infty}$  Analysis of Linear Systems with Jumps:  $_{\infty}$  Anarysis or Linear Systems with Jump<br>Applications to Sampled-Data Control  $^{\star}$ H∞ Applisoniane D 50 (2010) 150 115<br>21 Application of Linear Spectrum with Jumps:  $\mathcal{H}_{\infty}$  Analysis of Linear Systems with Jumps:  $\mathcal{H}_{\infty}$  Analysis of Linear Systems with Jumps:<br>Analysis to Sampled-Data Control  $\star$

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 $\mu_{\infty}$  analysis of a certain class of hyperatureal systems. The main results presented in this paper are embedded in the context of Riccati equations and convex optimisation. These results, together with the classic Small-Gain Theorem, can be applied to design state feedback controllers for sampled-data systems subject to time-delays. Abstract: This note focuses on  $\mathcal{H}_{\infty}$  analysis of a certain class of hybrid linear systems. The design state reedback controllers for sampled-data systems subject to time-delays. main resuits presented in this paper are embedded in the context of Kiccati equations and convex<br>optimisation. These results, together with the classic Small Cain Theorem, can be applied to optimisation. These results, together with the classic small-Gain Theorem, can be applied to

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*Keywords:* Hybrid systems, Sampled-data systems,  $\mathcal{H}_{\infty}$  control, Linear systems, LMI. *Keywords:* Hybrid systems, Sampled-data systems, H<sup>∞</sup> control, Linear systems, LMI. *Keywords:* Hybrid systems, Sampled-data systems, H<sup>∞</sup> control, Linear systems, LMI. *Keywords:* Hybrid systems, Sampled-data systems,  $\pi_{\infty}$  control, Linear systems, LMI.

#### 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION

Hybrid (or jump) linear systems are dynamic models that  $\frac{f_{\text{max}}}{f_{\text{max}}}$  (Goeden) includes of  $\frac{f_{\text{max}}}{f_{\text{max}}}$  and  $\frac{f_{\text{max}}}{f_{\text{max}}}$  combine both continuous and discrete-time behaviours in their formulation (Goebel et al., 2009). Specific classes of show to analyze the distribution (Suppose the anti-groom). Specific classes of hybrid systems that are of great practical interest comprise systems and divergence proceed in the comprise surface someone systems (Costa et al., 2013). The reader should refer to  $t_1$ , 2009; The reduct shock references in addition to (Lunze and  $t_1$ ); Shorten et al., 2009; Shock references in addition to (Lunze and Lamnabhi-Lagarrigue, 2009; Shorten et al., 2007) for an overview of important results for this class of systems. overview of important results for this class of systems. Hybrid (or jump) linear systems are dynamic models that  $\mathcal{H}$  $\overline{\phantom{a}}$ Hybrid (or jump) linear systems are dynamic models that their formulation (Goebel et al., 2009). Specific classes of hybrid systems that are of great practical interest comprise switched systems (Sun and Ge, 2005) and Markov jump systems (Costa et al., 2013). The reader should refer to the aforementioned references in addition to (Lunze and  $\frac{11.7 \text{ T}}{2000 \text{ C}}$ Lamnabhi-Lagarrigue, 2009; Shorten et al., 2007) for an overview of important results for this class of systems.

Additionally to the previously mentioned classic hybrid models, jump systems also provide a natural time-domain models, jump systems also provide a natural time-domain modelly,  $\frac{1}{2}$  comprehensive analysis of both in a comprehensive analysis of both in and filtering formulation of several sampled-data control and filtering problems. A comprehensive analysis of both jump and presents. It comprehensive dialysis of soon jump and sampled-data systems is done in (Ichicawa and Katayama, sampled-data systems is done in (Ichicawa and Katayama, 2001), where the authors present meaningful results based Foot), where the database problems inclaiming the basic season on boundary value problems and on Riccati equations. on boundary value problems and on Riccati equations.<br>There exist several references in the literature to date There exist several references in the interacture to date<br>that extend the basic results provided in (Ichicawa and recent developments obtained for such systems are based for the contract of the contract of the Katayama, 2001) in a broad range of directions. Important recent developments obtained for such systems are based provided in the reference of the main stability conditions on convex descriptions of the main stability conditions on convex descriptions of the main stability conditions<br>provided in that reference. In (Briat, 2013), the authors provide stability and state feedback stabilisation results for sampled-data systems. In (Geromel and Souza, 2015),<br>for sampled-data systems. In (Geromel and Souza, 2015), optimal  $\mathcal{H}_2$  and  $\mathcal{H}_{\infty}$  state feedback controllers are deoptimal  $\pi_2$  and  $\pi_\infty$  state recursion controllers are de-<br>vised solving convex optimisation problems. In (Souza and Geromel, 2015), stability and  $\mathcal{H}_2$  and  $\mathcal{H}_{\infty}$  performance exeronier, 2015), stability and  $\pi_2$  and  $\pi_\infty$  performance<br>necessary and sufficient convex conditions are also analweed. Similar conditions are also devised for switched linear ysed. systems in (Briat, 2015). Other important references that systems in (Briat, 2015). Other important references that Additionally to the previously mentioned classic hybrid models, jump systems also provide a natural time-domain formulation of several sampled-data control and filtering problems. A comprehensive analysis of both jump and 2001), where the authors present meaningful results based that extend the basic results provided in (Ichicawa and Katayama, 2001) in a broad range of directions. Important recent developments obtained for such systems are based provided in that reference. In (Briat, 2013), the authors provide stability and state recuback stabilisation results<br>for consulted data metasses. In (Concernational Corresponde) for sampled-data systems. In (Geromel and Souza, 2015), ysed. Similar conditions are also devised for switched linear systems in (Briat, 2015). Other important references that approach the sampled-data control design problem from a jump systems viewpoint are (Hara et al., 1994; Chen. and Francis, 1991; Kabamba. and Hara, 1993; Sun et al., 1993). and Francis, 1991; Kabamba. and Hara, 1993; Sun et al., 1993). 1993).  $T_{\text{1}}$  matrix  $\frac{1}{2}$  in this paper is the jump linear is the ju approach the sampled-data control design problem from a jump systems viewpoint are (Hara et al., 1994, Chen. and Francis, 1991; Kabamba. and Hara, 1993; Sun et al., 1993).

The main object of study in this paper is the jump linear system whose realisation is given by The main object of study in this paper is the jump linear system whose realisation is given by The main object of study in this paper is the jump linear system whose realisation is given by

$$
\mathcal{H} : \begin{cases} \dot{x}(t) = Ax(t) + Bw(t), \\ y(t) = Cx(t) + Dw(t), \\ x(t_k) = Kx(t_k^-) + Lw_d(k), \end{cases} (1)
$$

which is valid for all  $t \in [t_k, t_{k+1}), k \in \mathbb{N}$ . In this paper, we which is valid for all  $t \in [t_k, t_{k+1}), k \in \mathbb{N}$ . In this paper, we<br>assume the sequence of *jump instants*  $(t_k)_{k \in \mathbb{N}}$  is such that, for given  $t_0 \in \mathbb{R}$  and  $h \in \mathbb{R}_+ \setminus \{0\}$ ,  $t_{k+1} - t_k = h$ ,  $\forall k \in \mathbb{N}$ , for given  $t_0 \in \mathbb{R}$  and  $h \in \mathbb{R}_+ \setminus \{0\}$ ,  $t_{k+1} - t_k = h$ ,  $\forall k \in \mathbb{N}$ ,  $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$ Implying that the system jumps periodically and that  $\text{Zeno's phenomenon}$  is ruled out. Denoting  $\mathbb{T} := [t_0, \infty)$ , the continuous-time signals involved in (1) are the state<br>the continuous-time signals involved in (1) are the state<br> $\overline{R}$ the continuous-time signals involved in (1) are the state<br> $x : \mathbb{T} \to \mathbb{R}^{n_x}$ , the output  $y : \mathbb{T} \to \mathbb{R}^{n_y}$  and the input  $x : \mathbb{T} \to \mathbb{R}^{n_x}$ , the output  $y : \mathbb{T} \to \mathbb{R}^{n_y}$  and the input<br>  $w : \mathbb{T} \to \mathbb{R}^{n_w}$ ;  $w_d : \mathbb{N} \to \mathbb{R}^{n_w}$  is a discrete-time input that only acts on the jump equation; the matrix sextuple  $(A, B, C, D, K, L)$  is composed of real matrices of  $\text{complex}$  (1, 2, 0, 2, 1, 1, 2) as compacted of real methods of compatible dimensions. The system is assumed to evolve comparise uniensions. The system is assumed to evolve<br>from a given initial condition  $x(t_0^-) \in \mathbb{R}^{n_x}$ ; it is also  $\frac{1}{2}$  possible, however, to consider initial conditions at  $t_0$  and,<br>thus, initial states at  $t^-$ , are particular assessment that thus, initial states at  $t_0^-$  are particular cases such that  $\pi(t) = K_N(t-1)$  $x(t_0) = Kx(t_0^-).$  $\mathbf{u}(u_k) = \mathbf{h} x(t_k) + \mathbf{L} w_d(\kappa),$ <br>which is valid for all  $t \in [t_k, t_{k+1}), k \in \mathbb{N}$ . In this paper, we assume the sequence of *jump instants*  $(k_k)_{k\in\mathbb{N}}$  is such that, for given  $t_0 \in \mathbb{R}$  and  $h \in \mathbb{R}_+ \setminus \{0\}$ ,  $t_{k+1} - t_k = h$ ,  $\forall k \in \mathbb{N}$ , implying that the system jumps periodically and that implying that the system jumps periodically and that Zeno's phenomenon is ruled out. Denoting  $\mathbb{T} := [t_0, \infty)$ ,<br>the continuous-time signals involved in (1) are the state the continuous-time signals involved in (1) are the state<br> $\mathbb{R}$  $w : \mathbb{T} \to \mathbb{R}^{n_w}; w_d : \mathbb{N} \to \mathbb{R}^{n_w}$  is a discrete-time<br>input that only acts on the jump equation the matrix input that only acts on the jump equation; the matrix  $s$ extuple  $(A, D, C, D, K, L)$  is composed of real matrices of from a given initial condition  $x(t_0) \in \mathbb{R}^{n_x}$ ; it is also<br>possible however to consider initial conditions at  $t_0$  and

Our main goal is to extend the  $\mathcal{H}_{\infty}$  performance measure Our main goal is to extend the  $\pi_{\infty}$  performance measure<br>defined in (Geromel and Souza, 2015; Souza and Geromel, also are more than a sound, 2015), so that the conditions, 2015). Based on this result and on the Small-Gain Theo-Forty). Based on this result and on the simal data rideo tem, we also show how these contribute can be upphed to design sampled-data state-feedback controllers subject to to valid the main feature of the main features of the main feature shall be used the development of the main theoretical developments of the validate (and to point out the main features of) the main theoretical developments of this note. Our main goal is to extend the  $\mathcal{H}_{\infty}$  performance measure main theoretical developments of this note. 2015). Based on this result and on the Small-Gain Theorem, we also show how these conditions can be applied to design sampled-data state-feedback controllers subject to time-delays. An example from the merature shall be used<br>the sullate (and to maint ant the usein fectures of) the to validate (and to point out the main features of) the main theoretical developments of this note.

Notation. For real matrices and vectors,  $\binom{T}{k}$  indicates transpose. For square matrices,  $tr(\cdot)$  denotes the trace transpose. For square matrices,  $\mathbf{tr}(\cdot)$  denotes the trace<br>function. The sets of natural, real and nonnegative real function. The sets of natural, real and homegative real numbers are indicated by  $N$ ,  $\mathbb{R}$  and  $\mathbb{R}_+$ , respectively. The Euclidean norms adopted in this paper are denoted as follows; Usual norms adopted in this paper are denoted as follows;<br>The Euclidean norm of a vector x in  $\mathbb{R}^n$  is denoted by **Notation.** For real matrices and vectors,  $(')$  indicates numbers are indicated by  $\mathbb{N}$ ,  $\mathbb{R}$  and  $\mathbb{R}_+$ , respectively. The Euclidean norm of a vector x in  $\mathbb{R}^n$  is denoted by

 $\star$  This work was in part supported by Science Foundation Ireland nia wom was in part supported sy service roundation notation<br>(SFI) grant 11/PI/1177; Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP/Brazil) grant 2014/17074-0; Conselho Nade São Paulo (FAPESP/Brazil) grant 2014/17074-0; Conselho Na-<br>cional de Desenvolvimento Científico e Tecnológico (CNPq/Brazil) grant 303850/2014-0.  $\gamma$  This work was in part supported by Science Foundation Ireland cional de Desenvolvimento Científico e Tecnológico (CNPq/Brazil)

grant 303850/2014-0.<br>M. Souza was with the School of Electrical and Electronic Engineering, University College Dublin, Republic of Ireland, when parts of this work were developed. this work were developed. ing, University College Dublin, Republic of Ireland, when parts of this work were developed.

 $||x|| \triangleq \sqrt{x'x}$ ; the  $\mathcal{L}_2$  and  $\ell_2$  norms for signals are denoted by  $\|\cdot\|_2$ . For symmetric matrices, the symbol  $(\star)$  denotes each of its symmetric blocks. Finally,  $X > 0$   $(X \ge 0)$ denotes that the symmetric matrix  $X$  is positive definite (positive semidefinite); the set of all (positive definite) symmetric matrices of order *n* is denoted by  $(\mathbb{S}^n_+)$   $\mathbb{S}^n$ .

## 2. JUMP SYSTEMS: DYNAMICS

In this section, we present (and extend) classic results concerning the state dynamics and stability of the hybrid linear system H. We shall assume, for now, that  $w, w_d \equiv 0$ ; that is, the system is autonomous.

#### *2.1 State Dynamics*

As in any linear system, it suffices to find a *state transition matrix* associated with (1) to completely describe its dynamic behaviour. As stated in (Ichicawa and Katayama, 2001),  $\phi$  :  $\mathbb{T} \times \mathbb{T} \to \mathbb{R}^{n \times n}$  is a state transition matrix associated with (1) if, and only if,  $\phi$  satisfies

$$
\frac{\partial}{\partial t}\phi(t,s) = A\phi(t,s), \quad \forall t \in (t_k, t_{k+1}), k \in \mathbb{N}, \quad (2)
$$

$$
\begin{aligned}\n\phi(t_k, s) &= K\phi(t_k^-, s), \quad \forall k \in \mathbb{N}, \\
\phi(t, t) &= I.\n\end{aligned} \tag{3}
$$

In this case, it follows that any state trajectory  $x : \mathbb{T} \to$  $\mathbb{R}^{n_x}$  of H satisfies  $x(t) = \phi(t, s)x(s)$  for all  $s, t \in \mathbb{T}$ . Moreover, other properties such as  $\phi(t_k, t_k^-) = K$  and  $\phi(t, t_k^-) = \phi(t, t_k)K$ , for all  $t \geq t_k$ ,  $k \in \mathbb{N}$ , can also be verified. In the particular but important case in which the jump instants sequence is such that  $t_{k+1} - t_k = h > 0$ for all  $k \in \mathbb{N}$ , the state transition matrix also satisfies  $\phi(t+h, s+h) = \phi(t, s)$ , for all  $t, s \in \mathbb{T}$ ; see (Ichicawa and Katayama, 2001).

### *2.2 Stability*

We are now able to provide necessary and sufficient conditions for the state dynamics of  $H$ . First, we have to extend an important stability concept that is widely adopted in the LTI case (Rugh, 1996; Ichicawa and Katayama, 2001). *Definition 1.* (Exponential Stability). The hybrid linear system H is said to be *uniformly exponentially stable* if there exist finite positive constants  $\kappa$  and  $\alpha$  such that for any  $t_0 \in \mathbb{R}$  and  $x(t_0) \in \mathbb{R}^{n_x}$ , the corresponding solution  $x(\cdot)$  satisfies

$$
||x(t)|| \le \kappa e^{-\alpha(t-t_0)} ||x(t_0)||, \quad \forall t \in \mathbb{T}.
$$
 (5)

Alternatively, uniform exponential stability can be completely described by the state transition matrix (Rugh, 1996).

*Lemma 2.* The hybrid linear system  $H$  is uniformly exponentially stable if, and only if, there exist finite positive constants  $\kappa$  and  $\alpha$  such that

$$
\|\phi(t,s)\| \le \kappa e^{-\alpha(t-s)}\tag{6}
$$

for all  $t, s \in \mathbb{T}$  such that  $t \geq s$ .

There exist several stability conditions for the hybrid linear system  $H$  that is studied in this paper. We are particularly interested, however, in the necessary and sufficient inequality-based conditions presented in (Geromel and Souza, 2015); note that the proof of the necessity – that is, that (ii) implies (i) – is not presented in that reference and is included now.

*Theorem 3.* Consider the hybrid linear system  $H$ , whose realisation is given by (1), and let  $S \in \mathbb{S}_{+}^{n}$  and  $h \in \mathbb{R}_{+}^{\star}$  be given. The following statements are equivalent:

(i) For  $w \equiv 0$  and  $t_{k+1} - t_k = h$ ,  $\forall k \in \mathbb{N}$ ,  $\mathcal{H}$  is uniformly exponentially stable and the bound

$$
\int_{t_0}^{\infty} \|y(t)\|^2 dt < x(t_0^{\{-\}})^{\mathrm{T}} K^{\mathrm{T}} S K x(t_0^{\{-\}}) \tag{7}
$$

holds for any initial condition  $x(t_0^-) \in \mathbb{R}^n$ .

(ii) There exists a solution  $X : [0, h] \to \mathbb{S}^n_+$  to the boundary value problem defined by the linear differential equation

$$
\dot{X}(t) + A^{T} X(t) + X(t)A + C^{T} C = 0
$$
 (8)

and by the initial  $X(0) = S$  and final  $X(h) > K<sup>T</sup> S K$ conditions.

(iii) The linear matrix inequality

$$
A_h^{\mathrm{T}} K^{\mathrm{T}} S K A_h - S + C_h^{\mathrm{T}} C_h < 0 \tag{9}
$$

holds, where the matrix pair  $(A_h, C_h)$  is such that

$$
A_h = e^{hA}, \quad C_h^{\rm T} C_h = \int_0^h e^{tA^{\rm T}} C^{\rm T} C e^{tA} dt. \quad (10)
$$

*Proof:* We shall prove that  $(ii) \Rightarrow (iii) \Rightarrow (i) \Rightarrow (ii)$ . To this end, let us define the positive definite quadratic function

$$
v: \mathbb{R}_+ \to \mathbb{R},
$$
  
\n
$$
t \mapsto x(t)^{\mathrm{T}} P(t) x(t),
$$
\n(11)

in which  $P : \mathbb{T} \to \mathbb{S}_+^{n_x}$  is such that  $P(t) = X(t - t_k)$ for all  $t \in [t_k, t_{k+1}]$  and  $k \in \mathbb{N}$ ; such function is a *periodic extension* of X and, thus,  $P(t_k) = X(0)$  and  $P(t_{k+1}^-) = X(h)$  hold for all  $k \in \mathbb{N}$ .

First, let us assume that (ii) holds. Since (8) is linear, it is clear that

$$
X(t) = e^{-tA^{T}} X(0) e^{-tA} - \int_{0}^{t} e^{(\tau - t)A^{T}} C^{T} C e^{(\tau - t)A} d\tau
$$
\n(12)

is its unique solution; see Rugh (1996) for details. In particular, we may take  $t = h$  and use both boundary conditions to conclude that the LMI (10) is verified, implying that (iii) holds.

Now, assume that (iii) holds. From (9), there exists  $\alpha > 0$ such that

$$
e^{hA^{\mathrm{T}}}K^{\mathrm{T}}SKe^{hA} \le e^{-2\alpha h}S. \tag{13}
$$

This inequality implies that

$$
v(t_{k+1}) \le e^{-2\alpha(t_{k+1} - t_k)} v(t_k)
$$
\n(14)

holds for all 
$$
k \in \mathbb{N}
$$
. Additionally, take any  $c_0 \ge 1$  satisfying  
\n
$$
e^{t(A+\alpha I)^{\mathrm{T}}}X(t)e^{t(A+\alpha I)} \le c_0X(0), \quad \forall t \in [0, h), \quad (15)
$$

which always exists since the positive definite solution X exists and is bounded. Using (15), we can conclude that

$$
v(t) = x(t)^{\mathrm{T}} P(t)x(t)
$$
  
=  $x(t_k)^{\mathrm{T}} e^{(t-t_k)A^{\mathrm{T}}} X(t-t_k) e^{(t-t_k)A} x(t_k)$   
 $\leq c_0 e^{-2\alpha(t-t_k)} x(t_k)^{\mathrm{T}} P(t_k) x(t_k)$   
=  $c_0 e^{-2\alpha(t-t_k)} v(t_k)$  (16)

holds for all  $t \in [t_k, t_{k+1}), k \in \mathbb{N}$ . Since the boundary conditions also imply that  $v(t_k) < v(t_k^-)$  holds for all k,

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