Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnline CONFERENCE PAPER ARCHIVE

IFAC-PapersOnLine 49-23 (2016) 022–027

NMPC-based Trajectory Tracking and Collison Avoidance of Underactuated Vessels with Elliptical Ship Domain ⋆

Mohamed Abdelaal * **Martin Fränzle** * **Axel Hahn** *

* *Computer Science Department, University of Oldenburg, Ammerländer Heerstr. 114-118, 26129 Oldenburg, Germany. (e-mail: {mohamed.abdelaal, martin.fraenzle, axel.hahn}@uni-oldenburg.de).*

Abstract: This paper presents a Nonlinear Model Predictive Control (NMPC) for position and velocity tracking of underactuated surface vessels of elliptical ship domains with a collision avoidance scheme. A constrained nonlinear optimization problem is formulated to minimize vessel states' deviation from a time varying reference generated by a virtual vessel over a finite horizon. Linear constraints is imposed on input force and moment to be within the physical limits. Collision avoidance is represented as separation of each pair of the elliptic disks, representing the ship domain of our vessel and each encountered one, and is formulated as nonlinear time-varying constraint over the prediction horizon. A real-time C-code is generated using the ACADO toolkit and `qpOASES` solver with multiple shooting technique for discretization and Gauss-Newton iteration algorithm. This algorithm is computationally efficient, thus enabling real-time implementation of the proposed technique. MATLAB simulations are used to assess the validity of the proposed technique.

Keywords: Nonlinear model predictive control (NMPC), Collision avoidance, Trajectory tracking, Underactuated vessel, Autonomous vessels.

1. INTRODUCTION

Although there are many collision avoidance algorithms for autonomous surface vessels, most of them consider it as a planning problem independent from the motion controller; ignoring vessel dynamics. This might lead a high collision risk specially in dense traffic areas where vessels closely encounter each other. Integrating collision avoidance schemes into trajectory tracking controller ensures that the vessel tracks a time-parameterized reference trajectory efficiently while doing necessary accurate maneuvers to avoid colliding with nearby vessels or obstacles. Following the International Regulations for Preventing Collisions at Sea (COLREGs), published by the International Maritime Organization (IMO), leads to non contrary actions among the vessels. Considering the ship domain, defined as the area around the ship that should be free from other ships and obstacles, enhances the accuracy of the maneuvering especially for close range encounters.

Tam et al. (2009) stated in their review paper that all the reported studies on collision avoidance have either disregarded the regulations, employed specific databases or used different safety domain geometries to emulate COLREGS; and assumed a highly simplified version of the ship dynamic model. There are many techniques, developed afterwards, used for solving collision avoidance problem from the planning point of view and handle COLREGs. In Szlapczynski (2012), evolutionary approaches are used to find a safe and optimum trajectory of surface vessels employing the kinematic model. In Zhuo (2014), A fuzzy logic approach is used, where the collision avoidance is formulated as an optimization problem and solved using particle swarm algorithm. A^* searching algorithm is used in Ari et al. (2013) to find the shortest path between two given coordinates in the presence of obstacles. Although the aforementioned techniques demonstrate collision avoidance and handle COLREGs, they ignore the dynamics of the vessel.

Recently, integrating collision avoidance into the the control design has attracted many researchers. In Wang and Ding (2014), linear Model Predictive Control (MPC) is used for the tracking and formation problem of multiagent linear systems with collision avoidance as a constraint for the optimization problem. To make use of linear MPC well-established theory, Alrifaee et al. (2014) have presented collision avoidance scheme for networked vehicles by successively linearizing the nonlinear prediction model using Taylor series. In the robotics domain, Wang et al. (2015) have used MPC for obstacle avoidance of a space robot. However, they have either considered linear agents, simplified the nonlinearities, or handled only static obstacles.

Compared to different ship domains presented in Szlapczynski and Szlapczynska (2015), elliptic disk is the best polygon that represent the vessel safety envelop and facilitate mathematical formulation for collision avoidance purposes. As discussed in Do (2012), representing vessels' ship domain by circular disks results in a problem of the large conservative area defined as the difference between

⋆ This research is supported by the State of Lower Saxony as part of the project Critical Systems Engineering for Socio-Technical Systems (CSE).

10.1016/j.ifacol.2016.10.316

the areas enclosed by the circle and the ellipse. Other shapes are complex mathematically for NMPC formulation.

The main contribution of this paper is to remove the drawbacks of considering collision avoidance as a planning problem and ignoring the dynamics, and to consider the vessels' ship domain in the controller design. An NMPC-based trajectory tracking scheme of three-degree-of-freedom (3-DOF) nonlinear dynamic model of underactuated vessels, with embedded collision avoidance, is presented. Collision avoidance is formulated as nonlinear time-varying constraints to the NMPC optimization problem by reformulating the separation conditions of two moving elliptic disks. Constraint prioritization is used to follow COLREGs rules. This will lead to accurate reference tracking and reduce the collision risk between vessels.

2. PROBLEM FORMULATION

The surface vessel model has 6-DOF: surge, sway, yaw, heave, roll, and pitch, which can be simplified to motion in surge, sway, and yaw under the following assumptions mentioned in Chwa (2011):

(1) The heave, roll, and pitch modes induced by wind and currents are negligible.
(2) The inertia, added mass, and hydrodynamic damping matrices are diagonal.
(3) The available control variables are surge force and yaw moment.

Based on that, the 3-DOF model, as presented in Fossen (2011), will be:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g\boldsymbol{u} \tag{1}$$

Here, $\boldsymbol{x} = [x_p \; y_p \; \psi \; u \; \upsilon \; r]^T \in \Re^6$ is the state vector, $\boldsymbol{u} = [\tau_u \;\; \tau_r]^T \in \Re^2$ is the input vector, x_p and y_p are the positions, and ψ is the heading angle of the ship with respect to the earth-fixed frame, u and υ are longitudinal and transverse linear velocities in surge (body-fixed x_B) and sway (body-fixed y_B) directions respectively, r is the angular velocity in yaw around body-fixed z axis (see Fig. 1), $f(\cdot)$, and $g(\cdot)$ are continuous nonlinear functions in $\boldsymbol{u}$ and $f(\cdot)$ is locally Lipschitz in $\boldsymbol{x}$ that satisfies $f(0) = 0$,

$$f(\boldsymbol{x}) = \begin{bmatrix} u\cos(\psi) - \upsilon\sin(\psi) \\ u\sin(\psi) + \upsilon\cos(\psi) \\ r \\ \dfrac{m_2}{m_1}\upsilon r - \dfrac{d_1}{m_1}u \\ \dfrac{m_1}{m_2}ur - \dfrac{d_2}{m_2}\upsilon \\ \dfrac{(m_1 - m_2)}{m_3}u\upsilon - \dfrac{d_3}{m_3}r \end{bmatrix} \in \Re^6, \text{ and}$$

$$g_(\boldsymbol{x}) = \begin{bmatrix} 0 \; 0 \; 0 \; \dfrac{1}{m_1} & 0 & 0 \\ 0 \; 0 \; 0 \;\; 0 & 0 & \dfrac{1}{m_3} \end{bmatrix}^T \in \Re^6 \times \Re^2.$$

The parameters m_1, m_2, m_3 are the ship inertia including added mass effects, and d_1, d_2, d_3 are the hydrodynamic damping coefficients. A time-varying reference trajectory is generated by a virtual ship with the same dynamics as (1):

$$\dot{\boldsymbol{x}}_r = f(\boldsymbol{x}_r) + g\boldsymbol{\tau}_r \tag{2}$$

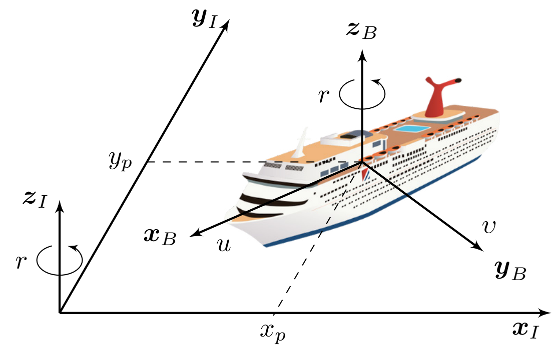

Fig. 1. Earth-fixed (x_I, y_I, z_I) and body-fixed (x_B, y_B, z_B) frames.

where $\boldsymbol{x}_r = [x_{pr} \; y_{pr} \; \psi_r \; u_r \; \upsilon_r \; r_r]^T$ denotes the generated reference states. The same assumptions as in Yan and Wang (2012) are adopted throughout this paper:

Assumption 1: All ship state variables (position, orientation, and velocities) are measurable or can be accurately estimated.

Assumption 2: The reference velocities and positions are smooth over time.

Hence, the control objective is to steer the vessel states $(x_p,\; y_p,\; \psi,\; u,\; \upsilon,\; r)$ to follow the reference states $(x_{pr},\; y_{pr},\; \psi_r,\; u_r,\; \upsilon_r,\; r_r)$ while satisfying control input and collision avoidance constraints considering the ship domain of the vessel by elliptic representation.

3. PRELIMINARIES

3.1 Separation Condition between Two Elliptic Disks

This subsection presents the conditions for two elliptic discs to be separated, that will be reformulated later for embedding into NMPC synthesis as collision avoidance constraint. Elliptic disks are the closed type of conic section results from the intersection of a cone by a plane, and is expressed in the plane with respect to inertial frame as (see Fig. 2):

$$\bar{\mathcal{A}} \equiv \{(x, y) \mid Ax^2 - 2Bxy + Cy^2 + (2By_p - 2Ax_p)x + (2Bx_p - 2Cy_p)y + (Ax_p^2 - 2Bx_p y_p + Cy_p^2 - 1) \leq 0\} \tag{3}$$

where $A = \left(\frac{cos(\psi)^2}{a^2} + \frac{sin(\psi)^2}{b^2}\right)$, $B = \frac{sin(2\psi)}{2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$, $C = \left(\frac{sin(\psi)^2}{a^2} + \frac{cos(\psi)^2}{b^2}\right)$, a and b are the radii of the ellipse, x_p and y_p are the position of the center of the ellipse, and ψ is the heading angle of the disk,. The elliptic disk can be represented by a 3×3 matrix $\mathcal{A} = [a_{i,j}]$ as:

$$\bar{\mathcal{A}} \equiv \{X \mid X^T \mathcal{A} X \leq 0\} \tag{4}$$

where $X = [x \; y \; 1]^T$ is the 3-D column vector containing the homogeneous coordinates and

$$\mathcal{A} = \begin{bmatrix} A & -B & By_p - Ax_p \\ -B & C & Bx_p - Cy_p \\ By_p - Ax_p & Bx_p - Cy_p & Ax_p^2 - 2Bx_p y_p + Cy_p^2 - 1 \end{bmatrix}.$$

By elementary math, the matrix $\mathcal{A}$ satisfies the condition that $det(\mathcal{A}) < 0$.

Theorem 1. Given two elliptic disks $(\bar{\mathcal{A}}, \bar{\mathcal{B}})$represented by the matrices $\mathcal{A} = [a_{i,j}]$ and $\mathcal{B} = [b_{i,j}]$ respectively: