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Some remarks on potential field constructions in a multi-obstacle environment

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Abstract: This paper addresses a novel combination between mixed-integer representations and potential field constructions for typical multi-agent marine control problems. First, we prove that for any kind of repulsive functions applied over a function which we denote as sum function, the feasible domain is piece-wise affine (PWA). Next, concepts like hyperplane arrangements together with potential field approaches are used for providing an efficient description of the feasible non-convex domain. This combination offers an original and beneficent computation of control laws under non-convex constraints. Simulation results over a common application of obstacle avoidance, which can be extended for unmanned surface vehicles, prove the effectiveness of the proposed approach.

Keywords: Obstacle avoidance, Potential field, Mixed integer programming, Autonomous vehicles.

1. INTRODUCTION

Autonomy is considered a key technology for continued growth in several maritime industries, see for instance examples in Grøtli et al. [2015]. Within the subsea oil- and gas, Remotely Operated Vehicles (ROVs) are currently being used for Inspection, Maintenance and Repair (IMR) operations. By reducing the total time of these operations, costs can be saved, as the support of expensive topside surface vessels is reduced. Underwater vehicles with autonomous functionalities that can carry out IMR more safely and efficiently than traditional ROVs, are therefore of great interest. Particularly interesting is the concept of *resident* vehicles, permanently located at the subsea facility, and which do not need surface support vessels at all.

Aquaculture is another industry where IMR operations need to be carried out regularly. One example of an application is the IMR of net cages at exposed fish farms. The integrity of the net is crucial to avoid fish escapees, and with expected move of fish farms to more exposed and remote locations, autonomous vehicles seems a necessity to replacement to human divers, Bjelland et al. [2015].

Finally, autonomy is expected to make its way into waterborne transport. The idea is that ships will be able to traverse their route completely without human presence on board. The dry bulk carrier has the greatest potential for becoming completely autonomous as it does not require much in terms of human supervision or intervention during the voyage, Rødseth and Burmeister [2012].

A common need for all of these systems, beyond the traditional functionalities of Guidance, Navigation and Control (GNC) systems, is the ability to detect and avoid collision, and replan around static and dynamic obstacles. Collision avoidance systems are typically divided into *deliberative* planning, *reactive* planning or a hybrid between the two, Tan et al. [2004]. Deliberative planning is used for planning to reach a long-term goal, where as reactive planning is suitable for real-time application where a fast response is required. A popular tool (in no small part due to it simplicity) is the potential field, where attractive forces guide the vehicles to the current target while repulsive forces keep it away from obstacles, Antonelli et al. [2001]. Commonly, these forces are generated using bell shaped functions, Antonelli et al. [2011], but more complex elements should be used to model complex geometry like harbors or coastlines, Pedersen and Fossen [2012].

In general, there are many control engineering problems where security regions need to be defined for dynamical systems moving in an environment with obstacles (which can be also defined by certain regions) so that a control objective is achieved Grundel et al. [2007]. Typical examples appear in robotics or multi-agent control in general, where issues like collision and obstacles avoidance are extensively studied (e.g., a ship needs to navigate with a safe passing distance between own ship and a target by indicating an exact danger area between own ship and the target ship) Wurman et al. [2008], Barnes et al. [2009], Xargay et al. [2013].

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The goal of this paper is to shed light on the use of hyperplane arrangements and polyhedral functions for efficiently developing repulsive potential fields that can be further used in control problems as tools for solving different objectives.

Since the feasible space of this type of problems is usually non-convex, a good candidate for efficient representations and availability of optimization solvers is represented by *Mixed-Integer-Programming* (MIP) Osiadacz [1990]. It has the ability to include non-convex constraints and discrete decisions in the optimization problem. However, despite the advantages previously mentioned, MIP has some numerical drawbacks (the search tree in a typical mixed-integer problem becomes exhaustive for relatively small size problems). Some efficient representations together with a reduction of the number of binary variables used in the problem formulation are detailed in Prodan et al. [2016], Stoican et al. [2014] with an application over obstacles avoidance and coverage problems.

Another candidate method for solving motion planning problems is the *artificial potential field* Khatib [1986]. To each element of interest (obstacle, other agent, destination and the like) a potential component is attached which combines to obtain the potential field. In turn, this field is used to provide control actions for the agent (usually some variation of the field gradient). Howard et al. [2002] uses a Potential Field-based method for mobile sensor network deployment. The fields are constructed such that each node is repelled by both obstacles and by other nodes, thereby forcing the network to spread itself throughout the environment. Jadbabaie et al. [2003] and Tanner et al. [2007] investigate the motions of vehicles to achieve a common velocity while avoiding collisions with obstacles and/or agents assumed to be points. Next, Roussos and Kyriakopoulos [2010] develop navigation functions which are then used to derive control laws for point-like agents with an associated disc of predefined radius around them. One shortcoming of this approach is the possible generation of traps (local minima). Relevant research on generating navigation functions that are free from local minima is available in the literature Rimon and Koditschek [1992]. However, generating a navigation function is computationally involved and thus not suitable for many navigation problems.

The present paper is motivated mainly by one of our previous work Prodan et al. [2013], where a predictive control strategy for trajectory tracking and decentralized navigation of multi-agent formations was presented. This work however, concentrates on the efficient construction of repulsive potential fields. More specifically, the original contributions are the following:

- we provide a generic framework for non point-like shapes which may define obstacles and/or safety regions around an agent;
- we consider the resulted repulsive potentials to generate a potential field;
- further, the field is considered in order to obtain (potentially constrained) control laws which govern an agent trajectory in a multi-obstacle environment.

The following notations will be used throughout the paper. Given a vector $v \in \mathbb{R}^n$, $\|v\|_\infty := \max_{i=1,\cdots,n} |v_i|$ denotes the infinity norm of v. Minkowski's addition of two sets $\mathcal{X}$ and $\mathcal{Y}$ is defined as $\mathcal{X} \oplus \mathcal{Y} = \{x + y : x \in \mathcal{X},\ y \in \mathcal{Y}\}$. The interior of a set S, $Int(S)$ is the set of all interior points of S. The collection of all possible n_c combinations of binary variables will be noted $\{0,1\}^{n_c} = \{(b_1, \ldots, b_{n_c}) : b_i \in \{0,1\}, \forall i = 1, \ldots, n_c\}$. Denote as $\mathbb{B}_p^n = \{x \in \mathbb{R}^n : \|x\|_p \leq 1\}$ the closed unit ball of norm p, where $\|x\|_p$ is the p-norm of vector x. Let $x_{k+1|k}$ denote the value of x at time instant $k+1$, predicted upon the information available at time $k \in \mathbb{N}$.

2. PRELIMINARIES

For collision avoidance problems in a multi-agent or multi-obstacle context the feasible region is non-convex. Usually, this region is considered as the complement of a (union of) convex region(s) which describes an obstacle and/or a safety region for an agent. Hereinafter, we introduce some tools and prerequisites which will be instrumental for potential field constructions.

2.1 Hyperplane arrangements

Consider a collection of N_o obstacles in $\mathbb{R}^n$ described as polytopic sets:

$$\mathbb{S} = \bigcup_{l=1}^{N_o} S_l, \tag{1}$$

for which the hyperplanes characterizing them are gathered into a finite collection $\mathbb{H} = \{\mathcal{H}_i\}_{i \in \mathcal{I}}$ from $\mathbb{R}^n$:

$$\mathcal{H}_i = \{x \in \mathbb{R}^n : h_i x = k_i\}, \ i \in \mathcal{I}, \tag{2}$$

with $\mathcal{I} \triangleq \{1 \ldots N\}$ and $(h_i, k_i) \in \mathbb{R}^{1\times n} \times \mathbb{R}$.

The hyperplanes in (2) partitions the space into two disjoint [1] regions represented as:

$$\mathcal{R}_i^+ = \{x \in \mathbb{R}^n : \quad h_i x \leq \quad k_i\}, \tag{3a}$$

$$\mathcal{R}_i^- = \{x \in \mathbb{R}^n : -h_i x \leq -k_i\}. \tag{3b}$$

Furthermore, these hypeplanes cut the space $\mathbb{R}^n$ into disjoint cells:

$$\mathcal{A}(\sigma) = \bigcap_{i \in \mathcal{I}} \mathcal{R}_i^{\sigma(i)}, \tag{4}$$

which are a feasible intersections of halfspaces (3a)–(3b) with the signs appropriately taken from the tuple $\sigma = (\sigma(1), \ldots, \sigma(N))$.

Then, there exists a sign tuple σ_l such that $S_l = \mathcal{A}(\sigma_l)$, i.e., can be written as an intersection [2] of regions (3a)–(3b).

Remark 1. Let the collection of all feasible tuples (which result in non-empty cells) be denoted by $\Sigma \subset \{-,+\}^N$. Then, collecting all tuples which describe obstacles from (1) we can denote the collection of interdicted tuples as $\bar{\Sigma} = \{\sigma \in \Sigma : \mathcal{A}(\sigma) \subseteq \mathbb{S}\}$ and the collection of available tuples as $\Sigma \setminus \bar{\Sigma}$. ♦

Such a partitioning of the space is called a hyperplane arrangement and is the union of all cells (4), that is, $\mathbb{R}^n = \mathbb{A}(\mathbb{H}) = \bigcup_{\sigma \in \Sigma} \mathcal{A}(\sigma)$.

[1] The relative interiors of these regions do not intersect, but their closures have as common boundary the affine subspace $\mathcal{H}_i$.

[2] We assume without loss of generality that each set S_l is characterized by a unique tuple.