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A Dynamic Model for Underwater Vehicle Maneuvering Near a Free Surface ⋆

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Abstract: This paper introduces a physics-based and control-oriented underwater vehicle model for near-surface operations. To construct the model, we follow an energy-based Lagrangian approach, where the presence of the free surface is incorporated using a *free surface Lagrangian.* This effectively modifies the system energy commonly used to derive the Kirchhoff equations, which govern underwater vehicle motion in an unbounded ideal fluid. The system Lagrangian is then used to derive the 6-DOF equations of motion for an underwater vehicle maneuvering near the free surface in otherwise calm seas. To illustrate the additional capabilities of the proposed model, we present an analytical hydrodynamic solution for a circular cylinder traveling parallel to the free surface. Comparisons are also drawn between the proposed model and the Cummins model (Cummins, 1962). While Cummins' model exactly satisfies the free surface boundary condition and approximately satisfies the body boundary condition, we choose to exactly satisfy the body boundary condition and approximately satisfy the free surface condition. This exchange removes the restriction that limits the Cummins equations to slow-maneuvering in a seaway.

Keywords: Underwater Vehicles; Control Oriented Models; Nonlinear Models;

1. INTRODUCTION

Low-dimensional dynamic models enable the design of effective guidance, navigation, and control systems for underwater vehicles (Fossen, 1994). These models traditionally neglect free surface effects since underwater vehicles typically operate well below the surface. When an underwater vehicle is tasked with maneuvering near the surface, however, these effects become unavoidable. A dynamic model which captures the missing physics could improve model-based control performance for scenarios including recovery operations and certain communications and sampling tasks. In comparison to an underwater vehicle that is deeply submerged, one operating near the free surface will radiate energy away as surface waves. Further, excitation forces due to incident surface waves can have a significant impact on the dynamics of an underwater vehicle operating in the wave affected zone.

For surface ships, Cummins (1962) devised a set of integro-differential equations to comprehensively capture the radiation forces. He proposed a hydrodynamic solution which satisfies the necessary boundary conditions for a vessel subject to small perturbations from a nominal course and heading. The equations incorporate the so-called *memory effects*, represented by a convolution integral that accounts for the effect of waves generated by past actions of the vessel. These equations were utilized by Bailey et al. (1998), where employing the Cummins framework enabled the unification of the linear (small perturbation) models for maneuvering and seakeeping. The model does not readily support control design, however, because the memory effects are not represented as ordinary differential equations. Kristiansen et al. (2005) employed model reduction techniques to approximate the memory effects using additional dynamic states. These model reduction results were implemented by Fossen (2005) and Perez (2005) to construct unified, nonlinear state space models. However, due to Cummins' original assumptions, the resulting model is limited to slow maneuvers in a seaway. This limitation is not an issue in certain, realistic scenarios, such as rudder roll stabilization (Perez, 2005). Interest remains, however, in developing a control-oriented unified model that accurately describes more aggressive maneuvering.

Fossen (1994) describes how a physics-oriented model for vessel motion is simplified for a deeply submerged underwater vehicle. Modeling efforts often begin with the Kirchhoff equations (Lamb, 1932), which exactly describe the motion of an underwater vehicle operating in an infinite domain of ideal fluid, where the fluid motion is due exclusively to the motion of the vehicle. Leonard (1997) showed that the Kirchhoff equations for a neutrally buoyant, underwater vehicle with offset centers of

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mass and buoyancy could be expressed in a non-canonical Hamiltonian form, a structure which is useful for nonlinear control design and stability analysis (Woolsey and Techy, 2009; Valentinis et al., 2015). Thomasson and Woolsey (2013) used Lagrangian mechanics to obtain a modified dynamic model that approximates the complex body-fluid interaction when an underwater vehicle is subject to a non-uniform and unsteady background flow. They demonstrated that the approximation compares well with analytical solutions for several special cases. This model was adapted to the case of an underactuated, underwater vehicle operating in monochromatic, plane progressive waves in (Battista et al., 2015). Force predictions obtained using the simplified model compared well with the analytical potential flow solution for a stationary, 2-D cylinder. The scenario of long-crested, irregular seas was analyzed in (Battista and Woolsey, 2015). The free surface is omitted from the analysis described in these papers, so the results fail to capture free surface effects.

This paper is intended to be a step toward constructing a nonlinear, parametric, control-oriented model for an underwater vehicle operating in waves under the free surface. The four conventional potential flow hydrodynamic forces that arise in these operating conditions are the Froude-Krylov, diffraction, added mass, and potential damping forces. The Froude-Krylov forces are incorporated using the Thomasson-Woolsey model (Battista et al., 2015; Battista and Woolsey, 2015). This work complements those results by using a first principles formulation of the *free surface Lagrangian* to augment the system energy used to derive Kirchhoff's equations. The present formulation neglects incident wave effects, but captures deviations in the added mass, radiative damping, and free surface suction that arise in free surface proximity operations.

The remainder of the paper is organized as follows. The system Lagrangian, including the free surface Lagrangian, is constructed in Section 2. The equations of motion are derived using a modified form of the Euler-Lagrange equations in Section 3. Some additional features of the equations of motion are explored using the simplified case of a 2-D cylinder in Section 4. Conclusions and future work are presented in Section 5.

2. DERIVING THE LAGRANGIAN

Prior to defining the Lagrangian, it is necessary to identify the relevant contributors to the system energy. When no free surface is present, Lamb (1932) defines the system as the combination of the underwater vehicle and the surrounding fluid continuum. For vessels operating at or near the free surface, this fluid boundary serves as a third contributor to account for the energy associated with free surface perturbations. We adopt this three-part system when constructing the Lagrangian, recognizing that the resulting model must capture the differences between surface ships and underwater vehicles operating near the free surface. For instance, in the extreme case where the vehicle floats on the surface, the free surface effects would be identical to those for surface ships. In the other extreme, where the vehicle is deeply submerged, the model must reduce to that described by Lamb (1932). Figure 1 depicts the system interactions for ships and subsea vessels. The greyed-out writing and dashed links between the free surface and the body emphasize the weaker connection between the two subsystems for underwater vehicles. To begin, we consider a system which consists of the body (b), a semi-infinite fluid volume (f), and a free surface (s), and define the system Lagrangian accordingly:

$$\mathcal{L} = \mathcal{L}_{\mathrm{b}} + \mathcal{L}_{\mathrm{f}} + \mathcal{L}_{\mathrm{s}}. \tag{1}$$

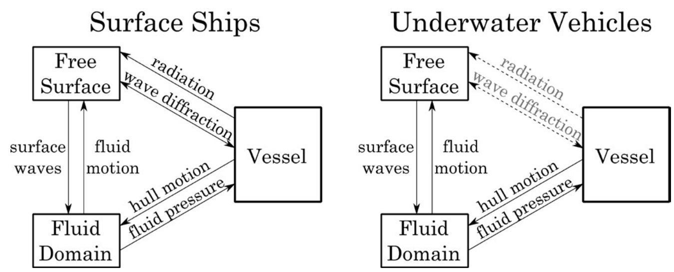

Fig. 1. The system energy storage devices, and mechanisms which transfer energy between them.

2.1 The Rigid Body Lagrangian

Consider an Earth-fixed frame with coordinate vectors $[\boldsymbol{i}_1\ \boldsymbol{i}_2\ \boldsymbol{i}_3]$, as shown in Figure 2, with reference point O in the unperturbed free surface, and the corresponding coordinates $\boldsymbol{x} = [x\ y\ z]^T$. A body-fixed frame at the body center of buoyancy B and coordinate vectors $[\boldsymbol{b}_1\ \boldsymbol{b}_2\ \boldsymbol{b}_3]$ coincide with the vehicle principal axes. Let $\bar{\boldsymbol{x}} = [\bar{x}\ \bar{y}\ \bar{z}]$ be the coordinates corresponding to the body axes. Let $\boldsymbol{x}_{\mathrm{b}}$ denote the location of B with respect to O, and define the orientation of the body axes in terms of the proper rotation matrix $\boldsymbol{R} \in SO(3)$. Then, let $\boldsymbol{\nu} = [\boldsymbol{v}^T \boldsymbol{\omega}^T]^T$ denote the body velocity, expressed in the body frame, consisting of the translational velocity $\boldsymbol{v} = [u\ v\ w]^T$ and the angular velocity $\boldsymbol{\omega} = [p\ q\ r]^T$. Let $\bar{\boldsymbol{x}}_{\mathrm{cm}}$ represent the location of the center of mass in the body frame. Then, the rigid body inertia matrix, expressed in the body frame, is given by [1]

$$\boldsymbol{M}_{\mathrm{b}} = \begin{pmatrix} \mathbb{M}_{\mathrm{b}} & -m\hat{\bar{\boldsymbol{x}}}_{\mathrm{cm}} \\ m\hat{\bar{\boldsymbol{x}}}_{\mathrm{cm}} & \mathbb{I}_{\mathrm{b}} \end{pmatrix}. \tag{2}$$

Following Leonard (1997), we define the rigid body's contribution to the system Lagrangian as follows

$$\mathcal{L}_{\mathrm{b}} = \frac{1}{2}\boldsymbol{\nu}^T \boldsymbol{M}_{\mathrm{b}} \boldsymbol{\nu} - mg\bar{\boldsymbol{x}}_{\mathrm{cm}} \cdot \boldsymbol{R}^T \boldsymbol{i}_3. \tag{3}$$

2.2 The Fluid Lagrangian

The rigid hull $\mathcal{B}$ acts as the boundary between the rigid body and the surrounding continuum of fluid particles $\mathcal{F}$. It is assumed that the fluid domain extends infinitely far in all directions except for one, which is bounded by the free surface $\mathcal{S}$. Potential flow theory allows the fluid velocity field $\boldsymbol{v}_{\mathrm{f}} = [u_{\mathrm{f}}\ v_{\mathrm{f}}\ w_{\mathrm{f}}]^T$ to be expressed in terms of a single scalar potential $\phi(\boldsymbol{x})$:

$$\boldsymbol{v}_{\mathrm{f}} = -\nabla\phi. \tag{4}$$

[1] The operator $\hat{\cdot}$ maps a three-vector to a 3×3 skew symmetric matrix satisfying $\hat{\boldsymbol{a}}\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$.