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# Verifying Collision Avoidance Behaviours for Unmanned Surface Vehicles using Probabilistic Model Checking

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**Abstract:** Collision avoidance is an essential safety requirement for unmanned surface vehicles (USVs). Normally, its practical verification is non-trivial, due to the stochastic behaviours of both the USVs and the intruders. This paper presents the probabilistic timed automata (PTAs) based formalism for three collision avoidance behaviours of USVs in uncertain dynamic environments, which are associated with the crossing situation in COLREGs. Steering right, acceleration, and deceleration are considered potential evasive manoeuvres. The state-of-the-art PRISM model checker is applied to analyse the underlying models. This work provides a framework and practical application of the probabilistic model checking for decision making in collision avoidance for USVs.



*Keywords:* unmanned surface vehicles; collision avoidance; probabilistic timed automata; formal methods; probabilistic model checking

## 1. INTRODUCTION

The development of unmanned surface vehicles spans several decades. The original radio-controlled vessels were designed for damage assessment and dangerous mine clearance operations. Over the past two decades, the development of more advanced sensors and the increased capabilities of computational power and communication technology coupled with a reduction in cost have motivated the use of USVs in novel applications and more complex missions such as minesweeping, environmental data collection and monitoring, water survey, anti-surface, and submarine warfare.

Collision avoidance (Savvaris et al. (2014)) is a central component for the design and development of USVs, due to that static obstacle or even dynamic intruders frequently exist in their paths. When several USVs or other vessels move in the same region, they act in fact as intruders to one another themselves. Thus, research on collision avoidance have become an active topic in the area of autonomous vehicles, and numerous algorithms have been proposed to realise the avoidance of static obstacles or dynamic intruders. In the past, the dynamic environments of USVs may be known in advance, since the intruders are assumed to have predefined or predicted moving behaviours. However, today's USVs commonly have to work in uncertain circumstances, where the movements of the intruders are not easy to be predicted accurately. Consequently, a number of probabilistic collision avoidance algorithms have been proposed in recent years, which models both the movements of the intruders and the operations of the vessels as probabilistic events.

The correctness of collision avoidance algorithms for USVs is very crucial. Simulation and testing have been the most frequently used analysis approach for verifying USVs' behaviours. However, either of them is by no means the best solution. Their weaknesses mainly lie in two aspects: (1) the results are incomplete, due to that only a subset of all the possible cases can be examined by physical system testing or software simulations; (2) the results are generally small sample data that are unsuitable for complex probabilistic analysis.

Formal verification now becomes a very useful alternative approach to traditional analysis approaches such as simulation and testing, because it is not only complete in logic and rigorous in mathematics but also adaptable for the description and analysis of probabilistic events. For example, probabilistic model checking is a quantitative verification approach widely applied in the reliability, safety, and performance analysis of both hardware and software systems. In general, there are three main phases involved in probabilisitic model checking: (1) a high level mathematical model is built to incorporate all the possible probabilistic behaviours; (2) formal logical formulae are derived to describe the key logical requirements; (3) an automatic tool such as PRISM is applied to check whether the mathematical model satisfies the logical requirements. If all the requirements are fully satisfied, the probabilistic behaviours are verified. Otherwise, it implies that some errors may exist in the original model. In recent years, formal verification has already been used to verify the path planning problem for autonomous vehicles (Quottrup et al. (2004); Fainekos et al. (2005)). However, there is little work that applied in verifying the collision avoidance problem in the same domain.

The aim of the paper is to formally verify three avoidance algorithms (steering, acceleration, and deceleration)


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that involve an USV and a single dynamic intruder. The paths of the USV and intruder cross each other, and their movements have both probabilistic and real-time properties, which is very suitable for using probabilistic timed automata. Thus, the probabilistic models and the logical formulae are first built, and then the PRISM model checker is applied to verify the underlying three avoidance strategies.

The rest of the paper is organised as follows. In Section 2, we present the collision avoidance algorithms for USVs. Section 3 introduces the necessary preliminaries of probabilistic timed automata and probabilistic model checking. Then, the PTAs are constructed in Sections 4 and 5 respectively. Finally, we conclude the paper in Section 6.

## 2. COLLISION AVOIDANCE FOR USVS

We have made some assumptions on the motion of the intruder vessel: (1) the whole moving process of the intruder can be divided into $n$ steps; (2) the intruder has a stepwise uniform motion, that is, it has different velocity at different time step, and the minimum and maximum velocities are denoted by $v_{min}$ and $v_{max}$; (3) the intruder changes the velocity at every time interval $\triangle T$; (4) the velocity for a time interval is constant, and independently and randomly selected within the range of $[v_{min}, v_{max}]$.

Based on the work of Miura and Shirai (2000), the probability distribution $p(x; n)$ for the intruder to reach the position $x$ after $n$ steps can be expressed as:

$$p(x; n) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-(\frac{(x-\bar{x}_i)^2}{2\sigma_i^2})^\gamma}, t \geq 0, \gamma, \alpha > 0 \quad (1)$$

where $\sigma^2 = \sigma_0^2 + n\sigma_{step}^2$, $\bar{x}_i = x_0 + n\bar{v}\triangle T$, and $\sigma_{step}^2 = (v_{max} - v_{min})^2/12$. Here, $x_0$, $\sigma$, and $\bar{v}$ are the initial position, variance, average velocity, respectively. Integrating $p(x; n)$ along the practical path of the intruder, one can obtain the probability of reaching the position $x$.

In this paper, three collision avoidance algorithms are given for a USV with only a single dynamic intruder. As shown in Fig. 1, the former and the latter are represented by a black-yellow USV and a white-blue ship, respectively. Assume that the paths of the USV and the intruder intersect at region $C$ with angle $\theta$ ($0° < \theta < 180°$) (see Fig. 1). $I_2$ and $U_2$ represent their positions when they begin to enter the collision region $C$, while $I_3$ and $U_4$ stand for those when they just leave such a region completely. $U_0$ is a reference position of the USV, which can be specified at an arbitrary point not over $U_2$. $I_0$ and $I_1$ are two reference positions of the intruder. $U_1$ is an undetermined position of the USV.

Under this condition, we mainly consider three avoidance behaviours: acceleration, deceleration, and steering. For the acceleration behaviour, the USV goes across region $C$ earlier than the obstacle does by increasing its velocity; for the deceleration case, it passes region $C$ later than the obstacle does by decreasing its velocity; for the steering one, it realises collision avoidance by changing its moving direction as shown in Fig. 2, where $\alpha$ is the heading angle and $\gamma$ is the turning radius. When $\alpha$ and $\gamma$ are given,

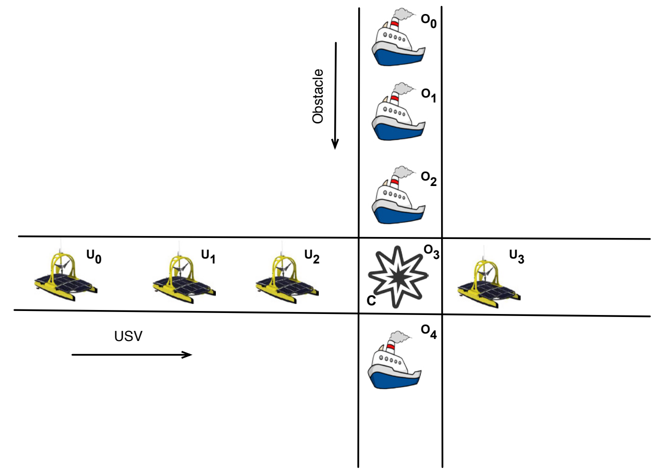


Fig. 1. The paths of an USV and an intruder: the USV is the stand-on vessel with respect to the International Regulations for Preventing Collisions at Sea 1972 (COLREGS).

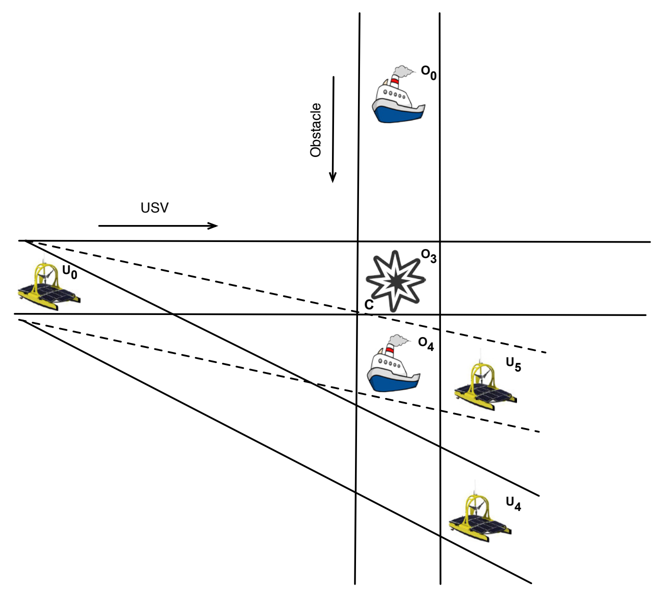


Fig. 2. The steering behaviour of the USV: the USV is the stand-on vessel with respect to the COLREGS.

according to its probabilistic behaviours, the USV may choose either $U_0\_U_4$ or $U_0\_U_5$. We represent $U_0\_U_4$ as the expected path, while $U_0\_U_4$ as the unexpected path. In Fig. 2, $U4$ represents the position where the obstacle begins to enter the path intersection region in the expected steering behaviour.

## 3. PROBABILISTIC MODEL CHECKING

### 3.1 Probabilistic Timed Automata

Full details about probabilistic timed automata (PTAs) can be found (Kwiatkowska et al. (2006); Norman et al. (2013)). We outline the important aspects in this section.

PTAs allow us to use the real-valued clocks of timed automata, together with the discrete probabilistic choice of MDPs. PTAs have real-valued clocks and, like MDPs