Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-23 (2016) 147–152

Comparison between Optimal Control Allocation with Mixed Quadratic & Linear Programming Techniques

Simone Grechi, Andrea Caiti

University of Pisa, Department of Information Engineering, Pisa, Italy.
Centro E. Piaggio, University of Pisa, Pisa, Italy
Interuniversity Center Integrated Systems for the Marine Environment (ISME), Italy
(e-mail: simone.grechi@for.unipi.it, andrea.caiti@unipi.it)

Abstract: The paper provides a comparison between different control allocation techniques in over-actuated Autonomous Underwater Vehicles. The pseudoinverse, Linear Programming (LP), Quadratic Programming (QP), Mixed Integer Linear Programming (MILP) and Mixed Integer Quadratic Programming (MIQP) are evaluated in simulation on the *V-Fides* vehicle model. The MILP and MIQP techniques allow to include in their implementations a more detailed characterization of the non-linear static behaviour of the actuators. This customizability can be also exploited to improve the practical stability of the system. The metrics used for comparison include the maximum attainable forces and torques, the integral of the error allocation and the required thrusters effort. Our simulation results show that, in particular with respect to thrusters effort, MILP and MIQP are the preferred allocation methods. The computational complexity associated to both methods is not such to compromise their implementation in operating vehicles; in particular, the MILP version is currently implemented in the *V-Fides* vehicle.

Keywords: Optimal Control allocation, Quadratic Programming, Linear Programming, Mixed-Integer Linear Programming, Mixed-Integer Quadratic Programming

1. INTRODUCTION

In the last decade improvements to control allocation methods have been proposed to achieve more accuracy, efficiency and effectiveness in control systems. Terrestrial, aerospace and marine vehicles can be over-actuated systems to improve manoeuvrability and to add flexibility and robustness. In the review paper, Johansen and Fossen (2013) and reference therein, the multidisciplinarity of control allocation problem is highlighted: from aerospace to underwater vehicles, solutions have been proposed and the cross-disciplinary transfer of ideas has been important for mutual progress in each sector. One of the advantages of over-actuated vehicles is their redundancy, allowing to a full or partial recovery if a fault on an actuator or effector occurs (Sarkar et al., 2002; Caiti et al., 2015). A modular design separating the control system from the control allocation allows an easy and portable implementation (Johansen and Fossen, 2013). Despite the advantages, sometime a complex resolution method must be implemented to solve the control allocation problem. In order to obtain a solution among the multiple ones that can result from redundancy, several methods based on the formulation of an optimal problem have been proposed in past years: from the classic approach, where unconstrained ℓ_2-norm minimum optimal problem is solved by *pseudoinverse* technique (Fossen and Sagatun, 1991), to more complex and customizable structures like Linear Programming (LP) and Quadratic Programming (QP) (Bodson, 2002; Enns, 1998; Bodson and Frost, 2011) where the force generated by actuators are considered linear regarding to commanded input. More computational demanding methods, based on Mixed-Integer formulations, can be considered to describe non-linear static characterization of the actuators. The main principle is to break in several points the non-linear function to obtain a piecewise linear form implementable into the optimal problem as linear constraints. In Bemporad and Morari (1999) is explained how the piecewise linear functions are implemented in Model Predictive Control framework, while in Bertsimas and Tsitsiklis (1997) a general formulation is exposed. A similar approach is implemented in Bolender and Doman (2004), where a Mixed Integer Linear Programming (MILP) optimal problem is described to solve the control allocation in a two-stage fashion, defining in each one a different cost functional. Moreover, a Mixed Integer Quadratic Programming (MIQP)-like formulation is proposed in Johansen et al. (2003) to solve the control allocation problem in marine vessels with rudder actuators, where the set of attainable thrust vectors is non-convex.

This paper presents a comparison between several control allocation techniques to evaluate the respective pros and cons as applied to an over-actuated Autonomous Underwater Vehicle (AUV). In particular pseudoinverse, LP, QP, MILP and MIQP are considered. But for the pseudoinverse, the other formulations exploit the customizability of the structure to improve the manoeuvrability and stability

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.335

of the vehicle. The MILP and MIQP are implemented including the dead-zone behaviour of the actuators static response into the optimal problem. The following metrics are used for comparison: the maximum attainable forces and torques, the integral of the error allocation and the required thrusters effort. The test case vehicle is the one developed within *V-Fides Project* and illustrated in Caiti et al. (2014). The vehicle simulator handles the hydrodynamic forces, kinematic equations, system control with allocation module and a detailed characteristic of actuators response.

The paper is organized as follow: in the next section the formal statement of the various optimal allocation problems is given. In Section 3 the main feature of the vehicle are described, the metric used for comparison is formally stated, and simulation results on a typical vehicle survey mission are reported. In last section results are discussed and conclusions are given.

2. PROBLEM STATEMENT

The control allocation problem can be defined as finding a set of input commands to the actuators such that the forces and torques exerted on manoeuvrable Degrees Of Freedom (DOFs) of the system are equal to the desired ones computed by the control module. In over-actuated systems the control allocation problem may admit multiple solutions due to the actuation redundancy. Therefore, several approaches were proposed in the past years to reformulate the control allocation into an optimal problem based on a proper cost function.

2.1 Pseudoinverse

In the simple and classic approach (Fossen and Sagatun, 1991), the allocation problem for over-actuated vehicles is reformulated as a ℓ_2-norm optimal problem, which can be formalized as:

$$\begin{aligned} \min_{f} \quad & f^T W f \\ \text{subject to } & \tau_d - Tf = 0 \end{aligned} \tag{1}$$

By defining as n the number of actuators and m the manoeuvrable DOFs, $f \in \mathbb{R}^n$ are the forces produced by the actuators and $W \in \mathbb{R}^{n\times n}$ is a weighting positive-definite matrix. The static transformation matrix $T \in \mathbb{R}^{m\times n}$ includes the information of the position and thrust axes of each actuator and it is employed to map the forces generated by each actuator in the total forces and torques exerted on the vehicle. Therefore, the objective of the optimal problem is to minimize the error between the desired generalized forces $\tau_d \in \mathbb{R}^m$ and the ones exerted on the vehicles. The solution to the minimization in (1), can be thus obtained via

$$\begin{aligned} T_w^\dagger &= W^{-1}T^T(TW^{-1}T^T)^{-1} \\ f &= T_w^\dagger \tau_d \end{aligned} \tag{2}$$

Note that the problem is unconstrained regarding the resulting forces, that means to have at disposal hypothetical unlimited forces from actuators. To achieve a feasible solution, the forces are chunked with the saturations imposed by the physical limitations of the actuators. A simple way to obtain the normalized input commands to the actuators, $u \in [-1, 1] \subset \mathbb{R}^n$, is to define the linear relation $f = Ku$, with $K \in \mathbb{R}^{n\times n}$ the diagonal matrix of the gains that characterize the static response of actuators.

$$u = K^{-1}T_w^\dagger \tau_d \tag{3}$$

A further practical approach can be adopted to introduce a more detailed static characteristic into the problem by introducing a look-up table downstream of the control allocation. Nevertheless, this formulation is somehow limiting since the only DOF available is the weighting matrix W.

2.2 Quadratic & Linear Programming

Control allocation can be cast as a minimization problem of the errors between the allocated and desired forces with respect to a chosen norm $\|\cdot\|_\ell$.

$$\min_{f} \| \tau_d - Tf \|_\ell \tag{4}$$

A first step to add more information about the actuators in the problem is to insert the saturation as constraints on the allocated forces. Choosing the ℓ_2-norm, the resulting optimal problem is solved with quadratic programming methods and can be formalized as following.

$$\begin{aligned} \min_{\alpha_s} \quad & \alpha_s^T H_s \alpha_s \\ & \tau_d - Tf = \alpha_s \\ & f_{min} < f < f_{max} \end{aligned} \tag{5}$$

Where $\alpha_s \in \mathbb{R}^m$ is the vector of the residuals, f_{min} and f_{max} the lower and upper bounds respectively imposed by the saturations and $H_s \in \mathbb{R}^{m\times m}$ is the definite positive weighting matrix. The formulation (5) takes into account only the minimization of the residuals, thus multiple suboptimal solutions may exist with respect to the forces, f. Therefore, in energy saving mindset the ℓ_2-norm of the allocated forces is evaluated in the cost functional.

$$\begin{aligned} \min_{f,\alpha_s} \quad & f^T H_f f + \alpha_s^T H_s \alpha_s \\ & \tau_d - Tf = \alpha_s \\ & f_{min} < f < f_{max} \end{aligned} \tag{6}$$

The $H_f \in \mathbb{R}^{m\times m}$ is the definite positive weighting matrix associated to the forces generated by the actuators. Observing the formulations (1) and (6), besides the addition of the saturations into the problem there is one more DOF in tuning perspective, H_s. In the later section will be shown how the correct tuning of H_s can improve the practical stability of the system by a judicious choice of the weighting matrix.

The minimization (4) can be cast as well as ℓ_1-norm and obtain a comparable formulation of (6). Thus, the sum of the quadratic residuals in the cost functional (6) are substituted as sum of the absolute values, $\sum_{i=1}^m K_{s_i} \mid \alpha_{s_i} \mid + \sum_{i=1}^n K_{f_i} \mid f_i \mid$, where $K_s^T \in \mathbb{R}^m$ and $K_f^T \in \mathbb{R}^n$ are the positive weighting vector of the residuals α_s and f, respectively. In (Boyd and Vandenberghe, 2004)