Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 196 (2007) 2713–2722
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Analytical and numerical investigation of strain-hardening viscoplastic thick-walled cylinders under internal pressure by using sequential limit analysis

S.-Y. Leu *

Department of Aviation Mechanical Engineering, China Institute of Technology, No. 200, Jhonghua Street, Hengshan Township, Hsinchu County 312, Taiwan, ROC

Received 26 November 2006; received in revised form 29 January 2007; accepted 5 February 2007

Abstract

Plastic limit load of strain-hardening viscoplastic thick-walled cylinders subjected to internal pressure is investigated numerically and analytically in the paper. The paper applies sequential limit analysis to deal with the quasi-static problem involving hardening material properties and weakening behavior corresponding to the strain-rate sensitivity and widening deformation. By sequential limit analysis, the paper treats the plasticity problems as a sequence of limit analysis problems stated in the upper bound formulation. Rigorous upper bounds are acquired iteratively through a computational optimization procedure with the internal pressure factor as the objective function. Especially, rigorous validation was conducted by numerical and analytical studies of thick-walled cylinders in terms of the plastic limit load as well as the onset of instability. It is found that the computed limit loads are rigorous upper bounds and agree very well with the analytical solutions.

Keywords: Sequential limit analysis; von Mises criterion; Nonlinear isotropic hardening; Viscoplasticity; Hölder inequality; Thick-walled cylinder

1. Introduction

Limit analysis features in capturing directly the important information for structural design or safety evaluation. Especially, it is considered to play the role of a snapshot look at the structural performance while providing the limit solution based on only simple input data (see, for example [16]). Thus, as well known, limit analysis is applied effectively to bound rigorously the asymptotic behavior of an elastic-plastic material by the lower bound or the upper bound theorem.

On the one hand, we can state theoretically the equality relation between the greatest lower bound and the least upper bound by duality (minimax) theorems as demonstrated by Yang [37,39,40,42], and Huh and Yang [16] mainly based on the generalized Hölder inequality [41]. On the other hand, it is numerically possible to enhance the accuracy of limit analysis and broaden its applicability to more complex problems in engineering applications as presented by Anderheggen and Knöpfel [1], Bottero et al. [3], Capsoni and Corradi [5], Christiansen [7], Corradi et al. [10], Dang Hung [11], Hodge and Belytschko [15], Pastor et al. [29], Sloan [34], Yang [40], Zhang et al. [43] by the use of finite element methods [33] together with mathematical programming techniques [25].

Furthermore, the fact that sequential limit analysis is an accurate and efficient tool for the large deformation analysis has been illustrated extensively by Corradi et al. [8], Corradi and Panzeri [9], Huh and Lee [17], Huh et al. [18], Hwan [19], Kim and Huh [22], Leu [26,27], Leu and Chen [28] and Yang [42]. By sequential limit analysis, a sequence of limit analysis problems is conducted sequentially with updating local yield criteria in addition to the

* Fax: +886 3 5936297.
E-mail address: syleu@cc.chit.edu.tw

0045-7825/$ - see front matter
doi:10.1016/j.cma.2007.02.001

configuration of the deforming structures. In each step and therefore the whole deforming process, rigorous upper bound or lower bound solutions are supposedly acquired sequentially as to bound the real limit solutions. Especially, a combined smoothing and successive approximation (CSSA) algorithm presented by Yang [38] has been utilized successfully with satisfactory results at a modest cost in certain problems of limit analysis by Huh and Yang [16] and sequential limit analysis by Huh and Lee [17], Huh et al. [18], Hwan [19], Kim and Huh [22], Leu [26,27], Leu and Chen [28] and Yang [42]. Particularly, some quantitative comparisons with elasto-plastic analysis have been made recently by Huh and his coworker [22]. As demonstrated in simulating the crashworthiness of structural members by Kim and Huh [22], only a fraction of the cost of elasto-plastic analysis was spent by sequential limit analysis. However, the unconditional convergence and numerical accuracy of the CSSA algorithm were demonstrated mostly by practical applications. Novelly, its convergence analysis was recently performed and validation was also conducted rigorously while extending the CSSA algorithm further to sequential limit analysis of viscoplasticity problems by Leu [26], or involving materials with nonlinear isotropic hardening by Leu [27].

In the literature, flow problems involving viscoplastic materials has been widely investigated using finite element methods (see, for example, [2,4,24,26,35]). On the other hand, the constitutive laws of viscoplastic materials have been often utilized to problems of regularized limit analysis (see, for example, [20,21,36]). In regularized limit analysis, the creep-strain rate is expected to converge to plastic-strain rates, see Jiang [20]. Based on the concept of sequential limit analysis, Leu [26] treated viscoplastic flow problems as a sequence of limit analysis problems. In each step of a deformation sequence, the limit load was computed by using the CSSA algorithm. Especially, the extended CSSA algorithm was shown to be unconditionally convergent by utilizing the Hölder inequality.

Leu [27] presented sequential limit analysis of plane-strain problems of the von Mises model with nonlinear isotropic hardening by using the CSSA algorithm. Particularly, the CSSA algorithm was proved to be unconditionally convergent by utilizing the Cauchy–Schwarz inequality in the work of Leu [27]. On the other hand, Leu and Chen [28] further applied the CSSA algorithm for sequential limit analysis involving nonlinear isotropic hardening materials to seek plastic limit angular velocity of rotating hollow cylinders.

Based on the previously successful applications in viscoplasticity [26] and nonlinear isotropic hardening problems [27,28], the paper aims to extend further the above-mentioned CSSA algorithm to upper-bound limit analysis considering the combination effect of strain hardening and viscoplasticity. Especially, numerical convergence of the CSSA algorithm is to be shown by means of the Hölder inequality. Particularly, the applicability of the CSSA algorithm is to be validated by numerical and analytical studies of thick-walled cylinders under internal pressure involving materials made of the von Mises model with viscoplastic nonlinear isotropic hardening. It is noted that such problems feature in involving hardening material properties and weakening behavior corresponding to the strain-rate sensitivity in addition to widening deformation. Novelly, a unified mathematical and numerical treatment of plasticity and viscoplasticity problems is to be established. And the limiting cases of the current work are to be converted to the previous results [26,27].

2. Problem formulation

We start with the statement of a plane-strain viscoplasticity problem of the von Mises model with nonlinear isotropic hardening. Naturally, the problem statement leads to the lower bound formulation. The corresponding upper bound formulation can be stated by duality theorems [37,39,40,42] mainly following the work of Huh and Yang [16]. As shown by Yang [37,39,40,42], Huh and Yang [16], the duality theorem theoretically equates the greatest lower bound to the least upper bound. Therefore, we can approach the real limit solution by maximizing the lower bound or by minimizing the upper bound.

2.1. Problem statement (lower bound formulation)

We consider the general plane-strain problem with the domain D consisting of the static boundary ∂D_s and the kinematic boundary ∂D_k [40]. The quasi-static problem is to seek the maximum allowable driving load under constraints of static and constitutive admissibility such that

$$\begin{aligned} &\text{maximize} \quad q(\sigma) \\ &\text{subject to} \\ &\quad \nabla\cdot\sigma = 0 \quad \text{in } D, \\ &\quad \sigma\cdot\vec{n} = q\,\vec{t} \quad \text{on } \partial D_s, \\ &\quad \|\sigma\|_v \leqslant H(\bar{\varepsilon},\dot{\bar{\varepsilon}}) \quad \text{in } D, \end{aligned} \tag{1}$$

where $\vec{n}$ indicates the unit outward normal vector of the boundary and the traction vector $\vec{t}$ is scalable distribution of the driving load on ∂D_s with the load factor q; $\|\sigma\|_v$ means the von Mises primal norm on stress tensor σ and the hardening function H is a function of the equivalent strain $\bar{\varepsilon}$ and the equivalent strain rate $\dot{\bar{\varepsilon}}$ describing viscoplastic strain-hardening. Therefore, this constrained problem is to sequentially maximize the load factor q representing the magnitude of the driving load for each step.

The primal problem (1) is the lower bound formulation seeking the extreme solution under constraints of static and constitutive admissibility. The statically admissible solutions satisfy the equilibrium equation and the static boundary condition. And the constitutive admissibility is stated by the yield criterion in an inequality form. We now refer to the work of Huh and Yang [16], Yang [42] to interpret the solutions as sets. First, the equilibrium equation is