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Distributed Nonlinear Control of a Plug-flow Reactor Under Saturation

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Abstract: This paper deals with the saturated control problem of a class of distributed systems which can be modelled by first-order hyperbolic partial differential equations (PDE). The objective is designing a distributed-parameter state feedback with guaranteed performance for this class of systems, using the Lyapunov stability theory and polynomial sum-of-squares (SOS) programming. For this, a polynomial parameter varying (PPV) model is employed to exactly represent the nonlinear PDE system in a local region of the state space and then, based on it, a PPV state-feedback law is designed guaranteeing exponential stability and actuator saturation in such region. The approach is illustrated here through the standard example of a nonisothermal plug-flow reactor.

Keywords: Hyperbolic PDE, Polynomial parameter varying, SOS, Saturation, Plug-flow reactor

1. INTRODUCTION

Numerous processes in industry including fluid heat exchangers, fiber spin lines or fixed-bed reactors, are essentially distributed in space, i.e. their behavior is determined not only by the time but also by the spatial position. Mathematical models for such systems can be obtained by applying the fundamental thermodynamic principles (balances of momentum, energy and material), resulting in a set of semilinear hyperbolic PDEs (Aksikas, 2005).

These systems usually do not work in isolation in the process and chemical industry, but work together as pieces of a larger equipment directly involved in production objectives. Therefore, stability analysis and control of such systems with guaranteed performance is of both theoretical and practical importance. As PDE systems are inherently infinite-dimensional, the existent control approaches for lumped-parameter systems (LPS) are hard to be used directly (Wang et al., 2011): actual controller implementations should be done within a finite number of actuators and sensors in practice. Thus, a guaranteed distributed-parameter controller design becomes a challenging task.

Many research works have been proposed for the control of PDE systems during the last decade. These methods can be divided into two well-known types: "indirect" and "direct" (Christofides, 2012). Indirect methods employ the original PDE model to design an infinite-dimensional controller (Ray, 1981), with its inherent difficulties, and it is then lumped for real implementation. Direct methods apply spatial discretization methods (e.g., finite differences, finite volume, orthogonal collocation or Galerkin's methods) to the PDE system in order to obtain an approximate model that contains a set of ordinary differential equations (ODEs) in time (Dochain et al., 1992). The subsequent ODE model is then used as the basis for the design of finite dimensional controllers. This approach benefits from the direct application of finite-dimensional control theory and methodologies but it has the important drawback of that the discretized ODE size may be very significant in order to reach the desired degree of approximation. This drawback causes the controller design to become high dimensional in structure and computationally complex.

The stabilization for tubular reactors in particular has been done typically using PIDs (despite the fact that such systems are nonlinear), provided suitable locations of sensors and actuators to ensure passivity (Alonso and Ydstie, 2001). PID designs are based on local linear models, obtained in different operation regions by input-output linearization (Aguilar et al., 2002; Mikhalevich et al., 2015). Though the use of PID controllers is quite simple, the main drawback arises in finding the right tuning in order to provide good robustness/performance trade-offs. In addition, even if a good trade-off has been achieved in practice, there is no guarantee of constraints satisfaction in the entire operating region. Recently, some works have given alternatives to the PID control of PDE systems. In particular, Aksikas et al. (2007) proposed an LQR control de-

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sign for nonisothermal PFRs using spectral factorization, and Wang et al. (2011) presented a fuzzy Takagi-Sugeno (TS) approach (quasi-LPV) for such systems based on linear matrix inequalities (LMI) and the well-known sector nonlinearity modelling (Tanaka and Wang, 2001). Nevertheless, there is still a common drawback for practical implementation of the above designs: the actuator's physical limitations are sometimes neglected or their treatment leads to very conservative controllers. However, treating them appropriately throughout the control design is key. There are, of course, other control strategies based in optimization (e.g., model predictive control) which explicitly consider input/state constraints, not discussed here.

Recently, TS/LPV modelling techniques have been extended to employ polynomial vertex models (Tanaka et al., 2009b) instead of linear ones. This class of polynomial parameter-varying (PPV) representation allows asymptotically reducing conservativeness of the design approach if the Taylor series decomposition is used for the sector modelling (Sala and Ariño, 2009). Then, the sum-of-squares (SOS) programming developed for pure polynomial systems (Papachristodoulou and Prajna, 2005; Pitarch et al., 2016b) is used to design PPV-based control systems with guaranteed performance (Tanaka et al., 2009a; Sala, 2009; Pitarch, 2013). Numerical solutions for these designs can be computed via the Gram-matrix decomposition and semidefinite programming (SDP) (Seiler, 2013).

The objective of this paper is extending the existent LPV control designs for a nonisothermal plug-flow reactor (PFR) to a PPV approach, including explicit consideration of the actuator limits from the design phase. First, a PPV-PDE model based on the Taylor series is proposed to accurately represent the nonlinear hyperbolic PDE system. Then, based on this model, a PPV state-feedback with antiwindup is proposed. In this way, the problem of finding suitable controller gains fulfilling input-saturation and guaranteeing local exponential stability of the closed-loop system is derived in terms of a set of spatially-dependent polynomial constraints, to be checked for SOS.

Briefly, the rest of the paper organizes as follows: Section 2 describes the PFR, its thermodynamic model and gives a PPV local representation for it; Section 3 states the control problem and its formulation into a SOS programming problem; Section 4 shows the effectiveness of the proposed approach with some results in simulation and, finally, a conclusion is drawn in the last section.

Notation: I stands for the identity. $D[v]$ denotes a diagonal matrix formed by the elements of v. $M^{[k]}$ will denote the k-th row of the matrix M. A symmetric matrix $P(x)$ in the spatial variable x is positive definite (semidefinite) in an interval $l_1 \leq x \leq l_2$ if $P(x) \succ 0$ ($P(x) \succeq 0$) for all $x \in [l_1, l_2]$. The symbol $(*)$ denotes the symmetric element in matrix expressions, e.g., $[M(x)+N(x)+(*)] \equiv [M(x)+N(x)+M^T(x)+N^T(x)]$. A SOS polynomial $p(y)$ in variables y is denoted by $p(y) \in \Sigma_y$. Similarly, an $n \times m$ SOS polynomial matrix $L(y)$ will be denoted by $L(y) \in \Sigma_y^{n\times m}$.

2. PLUG-FLOW REACTOR MODELLING

A nonisothermal PFR is an ideal flow reactor in which no back mixing occurs while a chemical reaction of the form $A \to \tilde{b}B$ takes place, being $\tilde{b} > 0$ the stoichiometric coefficient. Thus, the composition of the reaction mixture changes along the length x of the reactor, as represented in Figure 1. The reaction is endothermic and a jacket is used to heat the reactor, so that the system is dissipative, therefore open-loop stable.

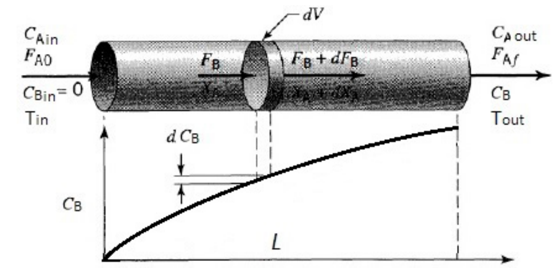

Fig. 1. Nonisothermal plug-flow reactor.

In Figure 1, C_A and C_B are the reactant and product concentrations respectively, T denotes the reactor temperature, $T_{in/out}$ and $C_{A,in/out}$ are defined as the temperature and concentration of the inlet/outlet streams respectively, F_B is the partial flow of product B, and L denotes the total length of the reactor. Under assumptions of perfect radial mixing, constant density and heat capacity of the reacting liquid, and negligible diffusive phenomena, a dynamic model of the process can be derived from material and energy balances. Note that C_B is known if C_A and T are known, so its mass balance has been omitted and only states T and C_A will be considered henceforth:

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial l} - \frac{k_0 \Delta H}{\rho_p C_p} C_A \cdot e^{-\frac{E}{RT}} + \frac{4h}{\rho_p C_p d}(T_J - T) \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -v\frac{\partial C_A}{\partial l} - k_0 C_A \cdot e^{-\frac{E}{RT}} \quad (2)$$

Where E, R, k_0, ΔH, h and d are the activation energy, the ideal gas, the pre-exponential factor, the enthalpy of the reaction, the wall heat transfer coefficient and the reactor diameter, respectively. The control input is chosen to be the spatially distributed jacket temperature T_J and t, l denote the independent time and space variables.

The process is subject to the boundary conditions

$$T(0,t) = T_{in}, \quad C_A(0,t) = C_{A,in}, \quad C_B(0,t) = 0 \quad (3)$$

and the initial state-conditions profiles are:

$$T(l,0) = T_0(l), \; C_A(l,0) = C_{A0}(l) \quad (4)$$

2.1 Nonlinear dimensionless PDE model

A scaled model will be obtained in order to compensate large differences in the magnitude orders of states, avoiding thus numerical problems in the SOS design phase. Hence, the following dimensionless states and input are introduced:

$$\chi_1 := \frac{T - T_{in}}{T_{in}}, \; \chi_2 := \frac{C_{A,in} - C_A}{C_{A,in}}, \; \phi_J := \frac{T_J - T_{in}}{T_{in}} \quad (5)$$

Define also $x := l/L$. Then an equivalent representation of (1)-(2) in variables (5) can be obtained (omitted for brevity [1]). Note that the dimensionless equilibrium profile ($t \to \infty$) in one variable can be computed given a prefixed

[1] The reader is referred to Aksikas (2005, Chap. 5) for the system description with more details.