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Port-Hamiltonian observer design for plasma profile estimation in tokamaks

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Abstract: This paper considers the problem of estimating magnetic and temperature profiles in tokamaks using nonlinear port-Hamiltonian observers. Two classes of observers preserving the port-Hamiltonian structure are considered: a proportional observer; and a proportional observer with an integral action. It is shown that the proposed passive observers are stable with respect to the interconnection of the observed system and the observer. For both designs, the observation gains are chosen such that the error dynamics takes the form of a port-Hamiltonian system. Simulation results for plasma profiles estimation illustrate the proposed observers performance, including in cases where their key physical parameters are badly known.

Keywords: Port-Hamiltonian systems, Nonlinear passive observers, Tokamaks, Distributed parameter systems, Profile estimation

1. INTRODUCTION

Controlled nuclear fusion can be viewed as a future promising sustainable source of energy. To ensure the fusion reaction, the fuel (isotopes of hydrogen) is heated up to 100 millions degrees. With this level of energy, the matter is a plasma (4th state of matter). A tokamak is a device where the plasma is magnetically confined. As depicted in Figure 1, three kinds of coils generate an insurmountable torus wall. An important problem in the operation of the tokamak is the estimation of state profiles, as the device is a distributed parameter system, and only discrete measurements are available to online the control system.

As defined in (van der Schaft and Jeltsema, 2014), port-Hamiltonian systems are embedded with a methodology to model multi-physic, nonlinear, multi-scale, and distributed parameter systems with Stokes–Dirac structures (Vu et al., 2016). The port-Hamiltonian framework is embedded with a set of control design techniques: passivity-based control; Casimir function; and energy- or power-shaping design (van der Schaft and Jeltsema, 2014). These control strategies assume complete knowledge of the state. However, in practice, and in particular for distributed parameter systems, the state cannot be measured entirely. In this framework, privileged outputs should satisfy the passivity property and this passive output should be easily measurable in practice. Nevertheless, the output is not enough to compute a robust control law, see for example (Vu et al., 2013). This motivates the development of port-Hamiltonian observers with the passive output as the only measurement.

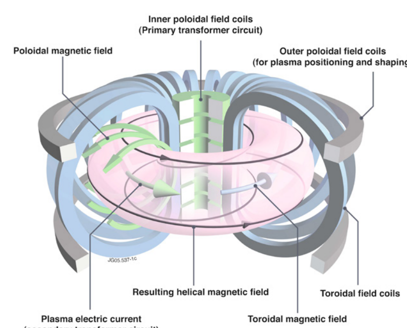

Fig. 1. Diagram of coils, magnetic fields and current in a tokamak (source: Euro-fusion).

Model-based observers, and more precisely structured ones, have been studied for different class of systems. For Lagrangian systems, Aghannan and Rouchon (2003) give an original approach to define a locally exponentially stable intrinsic observer. For port-Hamiltonian systems, the passivity property has been investigated. Wang et al. (2005) combine the controller and the observer in closed loop system while Venkatraman and van der Schaft (2010) propose a full order observer with a new input/output pairing to ensure passivity in the augmented system. This result is specific to mechanical systems and assumes perfect modeling of the observed system.

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In this paper, building on a contribution by Moreno (2008), we first present a proportional observer for the class of port-Hamiltonian systems. The passivity and stability properties of the augmented system, given by the system and its observer, are emphasized. Observation gains are computed such that the error dynamics takes the form of a port-Hamiltonian system. To improve the results given by the proportional observer, an augmented and robust version is proposed: a proportional integral observer. The integral state is added to the Hamiltonian energy function of the observer such that the system stays in the port-Hamiltonian framework. The robustness of this last observer is discussed. To motivate and illustrate the obtained observers, simulation results are given for the estimation of magnetic and thermal plasma profiles in tokamaks with the control model developed in (Vu et al., 2014).

The paper is organized as follows. In Section 2, the port-Hamiltonian framework is presented. Then, the proportional and the proportional integral observers are presented in Sections 3 and 4, respectively. Application of the discussed observers for the observation of the Tokamak and simulation results are presented in Section 5.

2. BACKGROUND: PORT-HAMILTONIAN SYSTEMS

Input-output formulation The port-Hamiltonian framework is energy-based and characterizes linear and nonlinear systems (van der Schaft and Jeltsema, 2014). The class of port-Hamiltonian systems is given by:

$$\begin{cases} \dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + gu \\ y = g^\top \dfrac{\partial H}{\partial x}(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector; $H(x) \in C^1(\mathbb{R}^n) \to \mathbb{R}$ represents the total stored energy in the system, also known as the Hamiltonian function; the conjugated port variables $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$, $m \leq n$ are the system inputs and outputs, respectively, and denote the exchanges between the system and the environment. The matrix $g \in \mathbb{R}^n \times \mathbb{R}^m$ maps the inputs; the matrices $J(x) = -J(x)^\top \in \mathbb{R}^n \times \mathbb{R}^n$ and $R(x) = R(x)^\top \geq 0 \in \mathbb{R}^n \times \mathbb{R}^n$ are called the interconnection and dissipative matrices, respectively. The skew-symmetric matrix $J(x) = -J^T(x)$ represents the exchanges of energy flow among the energy domains (with respect to a conservation law), while the symmetric matrix $R(x) = R^T(x)$ characterizes the flow responsible for the dissipative loss in the system (and is related to the irreversible entropy production). The output y is the passive output for this port-Hamiltonian system since the input/output pair satisfies the passivity property, defined below.

Energy balance, passivity and stability The storage energy function $H(x)$, differentiated along the trajectories of the system, leads to the following relation:

$$\dot{H}(x) = -\frac{\partial^\top H}{\partial x}(x)R(x)\frac{\partial H}{\partial x}(x) + y^\top u, \tag{2}$$

where the first term on the right hand side represents the internal dissipation due to the resistive elements and the second term is the variation due to the input/output flow. It can be deduced that the system (1) is said to be passive according to the input/output pair (u, y) if the following holds:

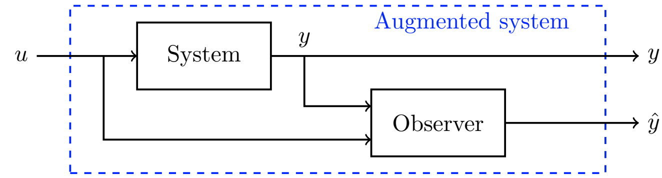

Fig. 2. Block diagram of the considered interconnection of the observed system and the observer.

$$\dot{H}(x) \leq y^\top u. \tag{3}$$

Furthermore, according to the result presented in (van der Schaft and Jeltsema, 2014), if the energy storage function $H(x)$ is bounded from below and the system is zero-state detectable (Byrnes et al., 1991), then the system (1) is stable. For the subclass of port-Hamiltonian systems with a quadratic energy function, as considered in the sequel, this property is satisfied.

3. NONLINEAR PROPORTIONAL OBSERVER DESIGN

We now turn our attention to nonlinear observers design using passivity theory, following the original contribution by Moreno (2008), specialized to the class of port-Hamiltonian systems introduced in the last Section.

3.1 Proportional observer

Definition 1. The dynamical system:

$$\begin{cases} \dot{\hat{x}} = [J(\hat{x}) - R(\hat{x})]\dfrac{\partial H}{\partial \hat{x}}(\hat{x}) + gu - L\Big(y - g^\top \dfrac{\partial H}{\partial \hat{x}}(\hat{x})\Big) \\ \hat{y} = g^\top \dfrac{\partial H}{\partial \hat{x}}(\hat{x}), \end{cases} \tag{4}$$

where $L \in \mathbb{R}^n \times \mathbb{R}^m$, is a passivity-based observer for the system presented in equation (1).

Definition 2. As depicted in Figure 2, the augmented system refers to the interconnection between the physical process and the observer, equations (1) and (4), respectively. The input/output pairs of this system are $((u, y), (u, \hat{y}))$.

The proposed observer mimics the system behavior and is driven by the control input u and the system output y pre-multiplied by the observation gain L.
The first result shows that the augmented system is passive and stable according to its input/output pairs.

Proposition 1. Consider the augmented system presented in definition 2. If there exists a matrix $L \in \mathbb{R}^n \times \mathbb{R}^m$ such that for all $(x, \hat{x}) \in \mathbb{R}^n$ the matrix

$$\begin{pmatrix} R(x) & -\dfrac{gL^\top}{2} \\ -\dfrac{Lg^\top}{2} & R(\hat{x}) + \dfrac{Lg^\top + gL^\top}{2} \end{pmatrix} \in \mathbb{R}^{2n} \times \mathbb{R}^{2n} \tag{5}$$

is positive semi-definite, then the augmented system is passive with respect to the input/output pairs $((u, y), (u, \hat{y}))$. Furthermore, if the Hamiltonian functions $H(x)$ and $H(\hat{x})$ are bounded from below for all $(x, \hat{x}) \in \mathbb{R}^n$, and if the augmented system is zero-state detectable, then the augmented system is stable.

Proof. The passivity property is deduced from the explicit formulation of the augmented system in a port-