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Synchronisation and Circuit Model of Fractional-Order Chaotic Systems with Time-Delay

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Abstract: The potential application of nonlinear systems and time delay, has a resulted in increasing attention to such systems. Studies on fractional-order nonlinear systems have also received attention. In this paper, we designed numerical and circuit models for synchronisation of fractional-order chaotic systems with time delay. The stability of the system's synchronisation was studied and synchronisation of the two circuits proposed. For different time delay values, the changes in the errors of synchronisation were examined. Finally the results of numerical simulations were compared with the results of circuit analysis to verify the theoretical analysis.

Keywords: Chaos, Synchronization, Fractional order systems, time delayed systems, fractional circuit model

1. INTRODUCTION

Although the fractional order (FO) calculus has been known since the seventeenth century, its application in engineering systems has only recently been realised by researchers (Miller and Ross 1973; Oustaloup et al. 2000). Modelling and simulation of the systems via FO calculus may be more realistic because they are generally fractional (El-Sayed et al. 2016; Xue, D. Chen, and Atherton 2007). FO systems are currently one of the hot topics in engineering. Researchers have generally focused on the stability of FO systems (El-Sayed et al. 2016), the modelling of electromagnetic waves (Xue, D. Chen, and Atherton 2007), FO chaotic systems and their synchronisation (Celik 2015; Radwana, et al. 2014; Gao et al. 2014; Matouk and Elsadany, 2014). The synchronisation of FO chaotic systems was first studied by Deng and Li (2005), FO chaotic systems can be applied particularly in secure communication (Atan, Türk and Tuntaş 2013).

Since chaotic systems are sensitive to initial conditions, they generate dissimilar signals in different initial conditions. Therefore, the purpose of synchronisation is to generate similar waveforms for two identical chaotic systems (these are called master and slave) (Pecora and Carrol 1990). Examples in the literature include projective synchronisation of time-delayed FO systems (Wang Yu and Wen 2014), synchronisation of FO chaotic systems with time-delay based on an adaptive fuzzy sliding mode (Lin and Lee 2013), and synchronisation of different FO time delay chaotic systems using active control (Tang 2014). These papers are generally focused on numerical analysis of the FO time delayed chaotic systems.

This paper is describes a numerical and a circuit model for FO chaotic systems and their synchronisation. Global stability for the synchronisation is defined according to Lyapunov stability criteria and according to the stability method, the input range is determined to make the system stable. The circuit model is designed according to the relevant parameters and control criteria are and numerical results compared with circuit simulation results.

The rest of this paper is organised as follows. In Section 2, chaos synchronisation of FO chaotic systems with time delay is introduced. Numerical analysis of FO chaotic systems and their synchronisations are given in Section 3. Section 4, describes a circuit design for FO chaotic systems with time delay and synchronisation, and presents simulation results. Section 5 conclusions.

2. CHAOS SYNCHRONISATION OF FO CHAOTIC SYSTEMS WITH TIME DELAY

2.1 Fractional Order Systems

FO systems are based on fractional calculus. Although FO calculus is old mathematical method, is still in wide use. FO systems employ many solution techniques. One of these techniques is the Grunwald-Letnikov (GL) method, which is used for discrete systems.

The Riemann-Liouville (RL) method is another technique used for continuous systems (Miller and Ross 1973; Oustaloup et al. 2000). The GL and RL equations are described by Eq. (1) and Eq. (2).

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to\infty} \frac{1}{\Gamma(\alpha)h^{\alpha}} \sum_{k=0}^{[(t-a)/h]} \frac{\Gamma(\alpha+k)}{\Gamma(k+1)} f(x-kh). \quad (1)$$

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau. \quad (2)$$

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Another technique is the Laplace transformation method. Laplace transformation can also be used for numerical solutions of FO systems (Miller and Ross 1973; Oustaloup et al. 2000). In zero initial conditions, the Laplace transforms of fractional derivation and integration are defined as follows:

$$L\left\{\frac{d^{\mu} f(t)}{dt^{\mu}}\right\} = s^{\mu} F(s), \mu \in R. \quad (3)$$

$$L\ I^{(\lambda)} f(t)\ = s^{-\lambda} F(s), \lambda \in R. \quad (4)$$

There are many approaches to solving s^{μ} and $s^{-\lambda}$ Laplace operators. Including the Crone, Carlson and Matsuda methods. The most widely used approximation of s^{μ} is Crone. Crone is a French acronym (non-integer order robust control-Commande Robuste d'Ordre Non-Entier) (Miller and Ross 1973; Oustaloup et al. 2000). In the Crone approximation, s^{μ} is calculated as follows:

$$s^{\mu} \approx C\prod_{a=1}^{N}\frac{1+\dfrac{s}{\omega_{z,a}}}{1+\dfrac{s}{\omega_{p,a}}}$$
$$\omega_{z,a} = \omega_l\left(\frac{\omega_h}{\omega_l}\right)^{\frac{(2a-1-\mu)}{2N}} \quad (5)$$
$$\omega_{p,a} = \omega_l\left(\frac{\omega_h}{\omega_l}\right)^{\frac{(2a-1+\mu)}{2N}}.$$

where C, ω_h and ω_l can be chosen as follows:

$$0 < \omega_l < \omega_h$$
$$C_0 > 0, \mu \in (0,1). \quad (6)$$

In FO systems, Laplace and Crone approximations are useful methods.

2.2 FO Chaotic Systems with Time Delay

The hyperbolic chaotic system, which is using hyperbolic tangent function (Ahmad and Sprott, 2003). Mathematical modelling of an hyperbolic chaotic system is given by

$$X'(\mathrm{t}) = AX(\mathrm{t}) - Bf(\mathrm{x}, \mathrm{t}, \tau). \quad (7)$$

$$f(\mathrm{x}, \mathrm{t}, \tau) = \sum_{j=-r}^{s} \tanh(x(\mathrm{t}-\tau) + jk). \quad (8)$$

where τ is time delay, and A and B are square matrix and vector matrix, respectively.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ k.a/2 \end{bmatrix}.$$

In the literature, the number of scrolls is dependent on r and s. If we choose r=s=0, a double scroll occurs in the system without time-delay (Ahmad and Sprott, 2003).

Similar to the classical hyperbolic chaotic system, the degree of the FO hyperbolic chaotic system can be any real number. The equation of FO hyperbolic chaotic system with time delayed is given by

$$X^{(\alpha)}(\mathrm{t}) = AX(\mathrm{t}) - Bf(\mathrm{x}, \mathrm{t}, \tau) \quad (9)$$

$$f(\mathrm{x}, \mathrm{t}, \tau) = \sum_{j=-r}^{s} \tanh(x(\mathrm{t}-\tau) + jk) \quad (10)$$

where τ is time-delay, and α_1, α_2 and α_3 are FOs. For a=b=c=0.8, r=s=0, k=10 and τ=0.1, the double scroll chaotic attractor is shown in Fig 1.

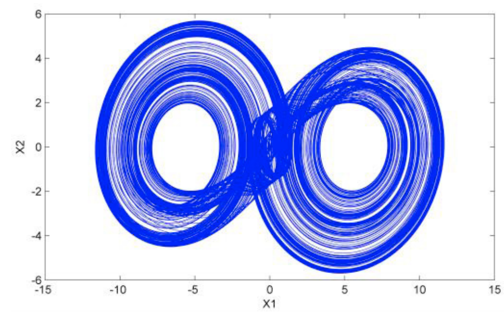

Fig. 1. Double scroll of chaotic system for a=b=c=0.8, r=s=0, k=10 and τ=0.1.

2.3 Synchronisation of the FO Chaotic Systems with Time Delay

In order to show the synchronisation of the systems with time delay, here we consider two FO chaotic systems named master and slave respectively. The master chaotic system is given as

$$\frac{d^{\alpha} x_m}{dt^{\alpha}} = y_m$$
$$\frac{d^{\alpha} y_m}{dt^{\alpha}} = z_m \quad (11)$$
$$\frac{d^{\alpha} z_m}{dt^{\alpha}} = ax_m + by_m + cz_m + \frac{a.k}{2}\tanh(x_m(\mathrm{t}-\tau)).$$

The slave chaotic system is given as

$$\frac{d^{\alpha} x_s}{dt^{\alpha}} = y_s + \mathrm{u}_1(\mathrm{t})$$
$$\frac{d^{\alpha} y_s}{dt^{\alpha}} = z_s + \mathrm{u}_2(\mathrm{t}) \quad (12)$$
$$\frac{d^{\alpha} z_s}{dt^{\alpha}} = ax_s + by_s + cz_s + \frac{a.k}{2}\tanh(x_s(\mathrm{t}-\tau)) + \mathrm{u}_3(\mathrm{t}).$$

where u(t) are control inputs and these inputs are dependent on synchronisation errors between the master and slave systems. The errors are given as

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_m - x_s \\ y_m - y_s \\ z_m - z_s \end{bmatrix}. \quad (13)$$

According to the errors, the control inputs of the system are

$$u_1(\mathrm{t}) = -\mathrm{e}_2(\mathrm{t}) - \mathrm{e}_1(\mathrm{t}). \quad (14a)$$