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Modelling, Analysis and Simulation of a Spatial Interaction Model

Tamara Vobruba*. Andreas Körner**
Felix Breitenecker***

** Institute of Analysis and Scientific Computing Vienna University of Technology, Vienna, Austria (e-mail: tamara.vobruba@tuwien.ac.at).*
*** Institute of Analysis and Scientific Computing Vienna University of Technology, Vienna, Austria (e-mail:andreas.koerner@tuwien.ac.at)*
**** Institute of Analysis and Scientific Computing Vienna University of Technology, Vienna, Austria (e-mail: felix.breitencker@tuwien.ac.at)*

Abstract: This paper reviews the theoretical backgrounds and formal structure of Gravity Models for Spatial Interaction Behaviour, as they build the foundation for the developed Migration Model. Furthermore the qualities and possibilities as well as the limits of these models are examined. Based on this a Migration Model is introduced, which is implemented to simulate Migration Movement into and within Europe. This can be used to analyse migration behaviour in general and in terms of influencing parameters.

Keywords: Qualitative Simulation, Mathematical Models, Probabilistic Models, Macro, Interaction Mechanism, Modelling, Simulation

1. INTRODUCTION

The Spatial Interaction Model is a macroscopic approach for describing any kind of spatial interaction behaviour between populations or regions. This Modelling approach has a wide range of applications, from the simulation of traffic flows, the movement of commuters or migrants, the trade with goods or the transmission of messages. (see Sen and Smith 2012, p. 1)

The best known and widely used type of Spatial Inter- action Models is the Gravity Model. The basic idea is the description of the Interaction between populations or regions with a relation referring to Newton's law of gravity. In geography and demography Gravity Models are used for a long time to analyse the flow of people, goods or capital. But often there is a lack of mathematical and theoretical foundation to understand the backgrounds. (see Dennett 2012, p. 2) So this work aims to review the theoretical backgrounds of Gravity Models and to develop a formal model description.

Based on this formalism a Migration Model is developed, which aims to describe international movements of migration. The structure of such a model, as well as the opportunities and the limits are examined. As part of this the model is methodologically analysed and characterised.

International migration movements can have a wide range of impacts and may influence political decisions and from national integration politics to international relations. In terms of analysing and understanding this movements modelling and simulation is of great importance. Beside the simulation of the movements itself also the investigation of influencing factors can be performed. Furthermore it is possible to make prognoses which are based on validated models and this can be the used for decision support.

Furthermore, this migration model is used to simulate the migration movement in the summer 2015 from Syria into and within Europe during the so called refugee crisis, to test this kind of models in terms of usability in a specific scenario.

2. SPATIAL INTERACTION MODEL

Spatial interaction is a movement or transmission over space, which is resulting of a decision process involving different influences. Interaction as physical movement is for example migration movement, where nonphysical movement could be the transmission of messages or the exchange of knowledge.

The Spatial Interaction Model is describing such spatial interaction with a relation, which is depending on different attributes. In the most general form, this model is given by the following equation.

$$I_{i,j} = f(A_i, R_j, C_{i,j}) \tag{1}$$

Here $I_{i,j}$ is a real number and is representing the interaction between the populations or regions i and j. It is described by a function of attributes A_i in i, attributes R_j in j and separation attributes $C_{i,j}$ between i and j.

The interaction is depending on a decision process which is happening on the basis of certain conditions. For example the decision of a commuter for a certain traffic route could be influenced by the costs of this route, the availability of public transport and the distance to work. This influencing factors are represented by the attributes A_i, R_j and $C_{i,j}$. Formally all the attribute are real numbers.

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In this general form more complexity can be reached by specifying the attributes and involving time dependencies.

3. GRAVITY MODEL

The widely used type of Spatial Interaction Models are the Gravity Models. Here the relation which is describing the interaction is based on Newton's law of gravity. This idea to draw analogies between physics and certain human behaviour has a long history. It came up in the year 1852 by Henry Charles Carey, who described the human migration behaviour as the "tendency to gravitate the fellow man".(Carey 1852, p. 42) This lead to examination of the relation between migration movement and the attraction of a regions, as well as the distance between regions. Ernest Charles Young was the first one that postulated the formal connection with the law of Gravity.(see Young 1924, p. 88) Investigating the movement of farm population he pointed out this coherence in the following formula.

$$M = k \cdot \frac{F}{D^2} \tag{2}$$

Here M is the absolute migration, F the intensity of attraction of a region, D the distance to this region and k a proportional constant. The dependency of attraction and the distance is based on Newton's law of gravity.

In the following years this approach was used to de- scribe human shopping behaviour and was refined by John Quincy Steward in the year 1941, who was developing the theory of demographic gravitation. (see Sen and Smith 1995,p. 3)

$$I_{i,j} = G \cdot \frac{P_i \cdot P_j}{d_{i,j}^2} \tag{3}$$

Here the influence of the fundamental physical law is obvious. The Interaction $I_{i,j}$ between the population centres i and j is direct proportional to the product of the so called population masses P_i and P_j, which are describing the attributes in i and j. Furthermore the interaction in indirect proportional to the squared distance $d_{i,j}$ between the population centres i and j. The constant G is called the demographic gravity constant.

A few years later in 1950 Steward developed a formula, which involves the possibility of different impacts of the attributes to the interaction. (see Sen and Smith 1995, p. 3)

$$I_{i,j} = G \cdot \frac{w_i P_i \cdot w_j P_j}{d_{i,j}^2} \tag{4}$$

Here w_i and w_j are the population weights. This weights are reflecting the heterogeneity of population masses and are treated as statistical parameters. With this parametric extension the model description steps back from the direct analogy to Newton's law, but it increases the flexibility of the mode la lot.

After the introduced forms of Gravity Models a lot of different formalisations were used. Ashish Sen and Tony Smith were introducing a general class of gravity models.(see Sen and Smith 1995, p. 4)

The general class of Gravity Models has the following form

$$I_{i,j} = A(i) \cdot B(j) \cdot F(d_{i,j}), \tag{5}$$

where the interaction $I_{i,j}$ is a real number and is resulting of the product of a weighted function $A(i)$ of attributes in i, a weighted function of the attributes $B(j)$ in j and function of separation attributes $F(d_{i,j})$ between i and j. Here the weighted functions itself are functions from a finite more dimensional vector space over the real numbers and are mapping into the set of real numbers. So it is possible to include a set of different attributes and there heterogeneity.

4. MIGRATION MODEL

4.1 Model Equations

Based on this ideas the aim is to introduce a migration model, which is fitting to the introduced class of gravity models. So the focus is on the migration behaviour between the regions of origin, the possible regions of destination and the transit regions. The heterogeneity of the attraction or repulsion of this regions influencing the migration behaviour should be included. Furthermore this attribute function should be treated as time dependent to observe structural changes in the migration behaviour over time. The Model description should also have the flexibility to describe migration movement between a list of different included regions in different structure and trough different routes.

First of all to find a formal way of describing the regions, these are represented by vertices of a graph, where the neighbouring countries are connected through edges.

The graph of migration movement is defined as a directed graph $G = (V, E)$, where the finite set of vertices $V(G) = \{v_1, v_2, \ldots, v_l\}$ is describing the different regions of interest and the finite set of edges $E(G) = \{e_1, e_2, \ldots, e_k\}$ with $e_k = \langle v_i, v_j \rangle$ is describing the geographical possibility to migrate from on region v_i to the other region v_j. In the adjacency matrix $A = (a_{i,k})_{i,j=1}^{l}$ of the graph the $a_{i,j} = 1$ if and only if there is the geographical possibility to migrate from i to j.

With this formalism it is possible to describe the time dependent interaction $I_{i,j}(t)$ from the region v_i to the region v_j with the following equations.