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Optimal Information Control in Cyber-Physical Systems

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Abstract: In this paper, we address optimal information control in cyber-physical systems. In particular, we study the optimal closed-loop policy for transmission of measurements of a stochastic dynamical system through a communication channel given estimation and communication costs. We develop a framework for optimizing an aggregate cost function that incorporates the estimation and the communication costs over a finite time horizon. We obtain the optimal closed-loop policy, and show that it can be expressed directly in terms of the value of information. In addition, we propose an approximation algorithm that yields a suboptimal closed-loop policy. Numerical and simulation results are presented for a simple system.

Keywords: Closed-Loop Policy, Estimation, Event-Driven Sampling, Value of Information

1. INTRODUCTION

Cyber-physical systems are tomorrow's systems in which cyber and physical components interact in all scales and levels. They deeply integrate computation, communication, and control into physical systems. In this paper, we address optimal information control in cyber-physical systems. Consider two agents Alice and Bob. Alice who has access to measurements of a stochastic dynamical system in the environment desires to inform optimally Bob whose task is to estimate the state of the system. However, the directed communication of Alice to Bob has a price due to the associated energy consumption, communication constraints, computational limits, and security issues. Therefore, Alice should devise a policy that transmits only data that is of *valuable information* to Bob. From an information theoretic perspective, Alice and Bob require a *source encoder* and a *source decoder*, respectively. Then, a challenging problem is to design an optimal encoding policy given estimation cost at the decoder and cost of the communication. This study has a broad range of applications including planetary exploration, environmental monitoring, wearable sensing, teleoperation, and many other examples of cyber-physical systems (Lee (2008)).

In this study, the encoder employs a sampler to control the information flow in the communication channel. *Nonuniform sampling* (Mark and Todd (1981)) and its important subclass *event-driven sampling* (Åström and Bernhardsson (2002)) in the context of the estimation problem have received early attention in the literature. Meier et al. (1967) extend the work of Kushner (1964) by looking at the measurement control problem subject to measurement cost and constraints, and by proposing dynamic programming (DP) and the gradient method as computational procedures. Åström and Bernhardsson (2002) show that event-driven sampling can outperform periodic sampling with respect to the estimation error of a scalar linear system under a sampling rate constraint. Rabi et al. (2012) study optimal event-driven sampling as a stopping time problem for a scalar system under a finite transmission budget constraint. Molin and Hirche (2012) investigate the optimal design for event-driven sampling in a scalar system with communication cost by considering a two-player problem. Sijs and Lazar (2012) study the estimation problem in event-driven sampling taking into account the implied knowledge when no measurement is transmitted. Furthermore, in the last few years several sampling policies have been proposed including ones based on the error between the current measurement and the last transmitted measurement (Otanez et al. (2002); Miskowicz (2006)), on the measurement innovation (Wu et al. (2013)), on the covariance of the estimation error (Trimpe and D'Andrea (2014); Soleymani et al. (2016b); Soleymani et al. (2016a)), and on the Kullback-Leibler divergence between the prior and the posterior conditional distributions (Marck and Sijs (2010)).

In the sense of Witsenhausen (1971), and Bar-Shalom and Tse (1974), we classify sampling policies based on the information pattern of the problem under study into: *open-loop policies*, *feedback policies*, and *closed-loop policies.* All previous works on nonuniform sampling for estimation are based on either open-loop policies (Kushner (1964); Meier et al. (1967); Trimpe and D'Andrea (2014); Soleymani et al. (2016b); Soleymani et al. (2016a)) or feedback policies (Otanez et al. (2002); Miskowicz (2006); Rabi et al. (2012); Molin and Hirche (2012); Sijs and Lazar (2012); Wu et al. (2013)). In this paper, for the first time, we study the optimal closed-loop sampling policy. The information pattern in our problem is characterized as follows:

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(1) The encoder has access to imperfect information, i.e., the encoder cannot access the state of the process,
(2) The estimator used at the decoder is causal, i.e., the estimator depends on past and present transmitted measurements,
(3) The estimator used at the decoder neglects the implied knowledge when no measurement is transmitted,
(4) The sampling policy is closed-loop, i.e., the policy takes into account past and present measurements and the statistics of measurements in the future.

We develop a framework for optimizing an aggregate cost function that incorporates the estimation and the communication costs over a finite time horizon given the aforementioned information pattern. We define the *value of information* (VOI) as "the maximum value that the encoder would be willing to pay for the transmission of a measurement". We show that the optimal closed-loop policy can be expressed directly in terms of the value of information. In addition, we propose an approximation algorithm that yields a suboptimal closed-loop policy.

The outline of the paper is as follows. After an introduction on notations, the information control problem is formulated in Section 2. In Section 3, we obtain the optimal information control and propose an approximation algorithm. We illustrate numerical and simulation results in Section 4. Finally, concluding remarks are made in Section 5.

1.1 Notations

In this paper, we represent an n dimensional vector with $x = [x_1, \dots, x_n]^T$ where x_i is its ith component. We write x^T to denote the transpose of the vector x. The identity matrix with dimension n is denoted by I_n. We use $C^\dagger$ to denote the Moore-Penrose inverse of the matrix C. We write $\delta_{kk'}$ to denote the Kronecker delta function. We write $p(x)$ to denote the probability distribution of the stochastic variable x. The expected value and the covariance of x are denoted by $\mathbb{E}[x]$ and $\mathbb{C}\mathrm{ov}[x]$, respectively. The normal distribution with mean μ and covariance Σ is denoted by $N(\mu, \Sigma)$. For matrices A and B, we write $A \succ 0$ and $B \succeq 0$ to mean that A and B are positive definite and positive semi-definite, respectively.

2. INFORMATION CONTROL PROBLEM

2.1 Dynamical System and Information Control

Consider a discrete-time dynamical system generated by the following linear state equation:

$$x_k = F x_{k-1} + w_{k-1}, \tag{1}$$

$$y_k = H x_k + v_k, \tag{2}$$

for $k = 1, 2, \dots$ where $x_k \in \mathbb{R}^n$ is the state of the system at time k, F is the state matrix, $w_k \in \mathbb{R}^n$ is a white noise sequence with zero mean and covariance $Q\delta_{kk'}$ where $Q \succ 0$, $y_k \in \mathbb{R}^p$ is the output of the system at time k, H is the output matrix, and $v_k \in \mathbb{R}^p$ is a white noise sequence with zero mean and covariance $R\delta_{kk'}$ where $R \succ 0$. It is assumed that the initial state x_0 is a Gaussian vector with zero mean and covariance P_0, and that x_0, w_k, and v_k are mutually independent.

Measurements of the system are available to a source encoder that samples measurements at times k_s for $s = 1, \dots, M$ where M is unknown. Samples are transmitted through a communication channel, and received by a source decoder. Through this study, the decoder assumes that measurements are never compromised.

Definition 1. (information control). The *information control* δ_k at time k is

$$\delta_k = \begin{cases} 1, & \text{if } \exists s : k = k_s, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\delta_0 = 0$.

A set of information controls $\pi = \{\delta_1, \dots, \delta_N\}$ is called an information control policy (or a sampling policy). In addition, a policy is closed-loop if it takes into account past and present measurements and the statistics of measurements in the future.

Definition 2. (encoder's information set). The *encoder's information set* is the σ-algebra generated by past and present measurements and past information controls, i.e.,

$$\mathcal{J}_k = \sigma\{y_l, \delta_{l-1} \mid l \leq k\}. \tag{4}$$

Remark 1. The information set $\mathcal{J}_k$ is available at time k to the encoder for decision making, i.e., $\delta_k = \delta_k(\mathcal{J}_k)$.

Definition 3. (decoder's information set). The *decoder's information set* is the σ-algebra generated by measurements transmitted to the decoder, i.e.,

$$\mathcal{I}_k = \sigma\{y_l \mid l \leq k, \delta_l = 1\}. \tag{5}$$

Remark 2. The information set $\mathcal{I}_k$ specifies the set of measurements available at time k for filtration at the decoder. Notice that the encoder can reconstruct the decoder's information set $\mathcal{I}_k$ from its information set $\mathcal{J}_k$, i.e., $\mathcal{I}_k \subset \mathcal{J}_k$.

The encoder's information set is autonomous, i.e.,

$$\mathcal{J}_k = \sigma\{\mathcal{J}_{k-1}, y_k, \delta_{k-1}\}. \tag{6}$$

However, the decoder's information set is a function of the information control, i.e., $\mathcal{I}_k = \mathcal{I}_k(\delta_k)$. In particular, we can write

$$\mathcal{I}_k(\delta_k) = \begin{cases} \sigma\{\mathcal{I}_{k-1}, y_k\}, & \text{if } \delta_k = 1, \\ \mathcal{I}_{k-1}, & \text{otherwise.} \end{cases} \tag{7}$$

2.2 Estimate Dynamics

Filtration at the decoder is based on the decoder's information set $\mathcal{I}_k(\delta_k)$. We assume that the estimator neglects the implied knowledge when no measurement is transmitted. Therefore, the conditional distribution $p(x_k|\mathcal{I}_k(\delta_k))$ is a Gaussian distribution. Define

$$\hat{x}_k = \mathbb{E}[x_k|\mathcal{I}_k(\delta_k)], \tag{8}$$

$$P_k = \mathbb{C}\mathrm{ov}[x_k|\mathcal{I}_k(\delta_k)]. \tag{9}$$

The conditional distribution $N(\hat{x}_k, P_k)$ evolves in time due to the system dynamics, and is updated at times k_s due to measurements.

Consider the transformation $I_k = P_k^{-1}$ where I_k is the Fisher information matrix (FIM) (Guo et al. (2012)). Following the Kolmogorov forward equation (Åström (2006)), the estimate and the FIM in the interval $(k_{s-1}, k_s^-]$ are propagated as

$$\hat{x}_k = F\hat{x}_{k-1}, \quad k \in (k_{s-1}, k_s^-], \tag{10}$$

$$I_k = (F I_{k-1}^{-1} F^T + Q)^{-1}, \quad k \in (k_{s-1}, k_s^-], \tag{11}$$