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Remote-state estimation with packet drop

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Abstract: In the remote estimation system, a transmitter observes a discrete-time symmetric countable state Markov process and decides to either transmit the current state of the Markov process or not transmit. The transmitted packet gets dropped in the communication channel with a probability ε. An estimator estimates the Markov process based on the received observations. When each transmission is costly, we characterize the minimum achievable cost of communication plus estimation error. When there is a constraint on the average number of transmissions, we characterize the minimum achievable estimation error. Transmission and estimation strategies that achieve these fundamental limits are also identified.

Keywords: Remote-state estimation, decentralized control, distortion-transmission function.

1. INTRODUCTION

Remote-state estimation refers to a scenario in which a sensor observes a stochastic process and determines whether or not to transmit each observation to a remote receiver. In this paper, we consider a model where the communication takes place over a TCP-like protocol; so either the transmitted packet is delivered without any error to the receiver or the packet is dropped.

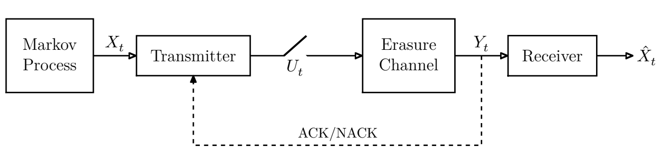

Fig. 1. The remote state estimation setup

Several variations of this setup has been considered in the literature. When the communication channel is ideal (i.e., there is no packet-drop), suboptimal and optimal transmission and estimation strategies are proposed in Imer and Basar (2005); Xu and Hespanha (2004); Lipsa and Martins (2011); Nayyar et al. (2013b); Molin and Hirche (2012); Chakravorty and Mahajan (2017). When there are packet drops, Li et al. (2013) consider the case when the transmitter can transmit only a fixed number of times, Xiaoqiang et al. (2016) consider the case when the probability of the packet-drop depends on the transmission power. Shi and Xie (2012) considers a similar setup with two energy levels and Dey et al. (2013) consider the case when the transmissions are noisy.

In this paper, we characterize the structure of optimal communication strategies as well as two fundamental trade-offs between communication and estimation: first when communication is costly and second when there is a constraint on the number of communications. In both cases, we identify communication strategies that achieve the optimal trade-offs.

2. PROBLEM FORMULATION

2.1 Remote estimation model

Consider the remote estimation setup shown in Fig. 1. A sensor observes a first-order time-homogeneous Markov process $\{X_t\}_{t\ge 0}$ with initial state $X_0 = 0$ and for $t \ge 0$,

$$X_{t+1} = aX_t + W_t, \tag{1}$$

where $\{W_t\}_{t\ge 0}$ is an i.i.d. innovations process. For simplicity, in this paper we restrict attention to $a, X_t, W_t \in \mathbb{Z}$. The results extend naturally to the case when $a, W_t, X_t \in \mathbb{R}$. We assume that W_t is distributed according to a unimodal and symmetric probability mass function p, i.e., for all $e \in \mathbb{Z}_{\ge 0}$, $p_e = p_{-e}$ and $p_e \ge p_{e+1}$. To avoid the trivial case, we assume $p_0 < 1$.

After observing X_t, the sensor decides whether or not to transmit the current state. This decision is denoted by $U_t \in \{0, 1\}$, where $U_t = 0$ denotes *no transmission* and $U_t = 1$ denotes *transmission*.

If the transmitter decides to transmit (i.e., $U_t = 1$), X_t is transmitted over a wireless erasure channel and there is a probability $\varepsilon \in (0, 1)$ that the transmitted packet is dropped. Let $H_t \in \{0, 1\}$ denote the state of the channel at time t. $H_t = 0$ denotes that the channel is in the *OFF* state and a transmitted packet will be dropped; $H_t = 1$ denotes that channel is in the *ON* state and a transmitted packet will be received. We assume that $\{H_t\}_{t\ge 0}$ is an i.i.d. process with $\mathbb{P}(H_t = 0) = \varepsilon$. Moreover, $\{H_t\}_{t\ge 0}$ is independent of $\{X_t\}_{t\ge 0}$.

Transmission takes place using a TCP-like protocol, so there is an acknowledgment from the receiver to the transmitter when a packet is received successfully. This means that the transmitter observes $K_t = U_t H_t$, which indicates whether the packet was successfully received by the receiver ($K_t = 1$) or not ($K_t = 0$).

The received symbol, which is denoted by Y_t, is given by

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$$Y_t = \begin{cases} X_t, & \text{if } K_t = 1 \\ \mathfrak{E}, & \text{if } K_t = 0, \end{cases} \tag{2}$$

where $Y_t = \mathfrak{E}$ denotes that no packet was received. Note that by observing K_t, the transmitter can compute Y_t. The transmitter uses this information to decide whether or not to transmit. In particular,

$$U_t = f_t(X_{0:t}, Y_{0:t-1}), \tag{3}$$

where $X_{0:t}$ and $Y_{0:t-1}$ are short-hand notations for $(X_0, \dots, X_t)$ and $(Y_0, \dots, Y_{t-1})$. The collection $f := \{f_t\}_{t\ge 0}$ of decision rules is called the *transmission strategy.*

After observing Y_t, the receiver generates an estimate $\{\hat{X}_t\}_{t\ge0}$, $\hat{X}_t \in \mathbb{Z}$, using an *estimation strategy* $g := \{g_t\}_{t\ge0}$, i.e.,

$$\hat{X}_t = g_t(Y_{0:t}). \tag{4}$$

The fidelity of estimation is measured by a per-step distortion $d(X_t - \hat{X}_t)$. We assume that:

- $d(0) = 0$ and for $e \neq 0$, $d(e) \neq 0$
- $d(\cdot)$ is even, i.e., $d(e) = d(-e)$
- $d(e)$ is increasing for $e \in \mathbb{Z}_{\ge 0}$.

2.2 The optimization problems

We are interested in two performance measures: expected total distortion and expected total number of transmission. Given any finite horizon strategy (f, g) for horizon T, the expected distortion is defined as

$$D_T(f, g) := \mathbb{E}^{(f,g)}\Big[\sum_{t=0}^{T} d(X_t - \hat{X}_t) \;\Big|\; X_0 = 0\Big]$$

and the expected number of transmissions is defined as

$$N_T(f, g) := \mathbb{E}^{(f,g)}\Big[\sum_{t=0}^{T} U_t \;\Big|\; X_0 = 0\Big].$$

Given any infinite horizon strategy (f, g) for discount factor β, $\beta \in (0, 1)$, the expected distortion is defined as

$$D_\beta(f, g) := (1-\beta)\mathbb{E}^{(f,g)}\Big[\sum_{t=0}^{\infty} \beta^t d(X_t - \hat{X}_t) \;\Big|\; X_0 = 0\Big]$$

and the expected number of transmissions is defined as

$$N_\beta(f, g) := (1-\beta)\mathbb{E}^{(f,g)}\Big[\sum_{t=0}^{\infty} \beta^t U_t \;\Big|\; X_0 = 0\Big].$$

We are interested in the following three optimization problems:

Problem 1 *(Costly communication, finite-horizon) In the model of Sec. 2.1, given a communication cost $\lambda \in \mathbb{R}_{>0}$ and a horizon T, find a transmission and estimation strategy (f^*, g^*) such that*

$$C_T^*(\lambda) := C_T(f^*, g^*; \lambda) = \inf_{(f,g)} C_T(f, g; \lambda), \tag{5}$$

where $C_T(f, g; \lambda) := D_T(f, g) + \lambda N_T(f, g)$ is the total communication cost and the infimum in (5) *is taken over all history-dependent strategies of the form* (3) *and* (4).

Problem 2 *(Costly communication, infinite-horizon) In the model of Sec. 2.1, given a discount factor $\beta \in (0, 1)$ and a communication cost $\lambda \in \mathbb{R}_{>0}$, find a transmission and estimation strategy (f^*, g^*) such that*

$$C_\beta^*(\lambda) := C_\beta(f^*, g^*; \lambda) = \inf_{(f,g)} C_\beta(f, g; \lambda), \tag{6}$$

where $C_\beta(f, g; \lambda) := D_\beta(f, g) + \lambda N_\beta(f, g)$ is the total communication cost and the infimum in (6) *is taken over all history-dependent strategies of the form* (3) *and* (4).

Problem 3 *(Constrained communication) In the model of Sec. 2.1, given a discount factor $\beta \in (0, 1)$ and a constraint $\alpha \in (0, 1)$, find a transmission and estimation strategy (f^*, g^*) such that*

$$D_\beta^*(\alpha) := D_\beta(f^*, g^*) = \inf_{(f,g): N_\beta(f,g) \le \alpha} D_\beta(f, g), \tag{7}$$

where the infimum is taken over all history-dependent strategies of the form (3) *and* (4).

Problems 1–3 are decentralized control problems. The system has two controllers or *agents*—the transmitter and the receiver—who have access to different information. In particular, the transmitter at time t has access to $(X_{0:t}, Y_{0:t-1})$ while the receiver at time t has access to $Y_{0:t}$. These two agents need to cooperate to minimize a common cost function given by (5), (6), or (7). Such decentralized control problems are investigated using *team theory* Mahajan et al. (2012).

In this paper we use the *person-by-person* approach in tandem with the *common information* approach to identify information states for both agents and obtain a dynamic programming decomposition. Then we use a partial order based on majorization to identify the structure of optimal transmission strategy. In particular, we show that optimal estimation strategy is similar to Kalman filtering and optimal transmission strategy is threshold-based. For Problems 2 and 3, we use ideas from renewal theory and constrained optimization to identify the optimal thresholds.

3. STRUCTURE OF OPTIMAL STRATEGIES

3.1 Person-by-person approach to remove irrelevant information at the transmitter

Proposition 1 *In Problem 1, there is no loss of optimality to restrict attention to transmission strategies of the form:*

$$U_t = f_t(X_t, Y_{0:t-1}).$$

Proof Arbitrarily fix the estimation strategy g and consider the *best response* strategy at the transmitter. Similar to the argument given in Witsenhausen (1979); Teneketzis (2006), it can be shown that $(X_t, Y_{0:t-1})$ is an information state at the transmitter and therefore the result of proposition follows from Markov decision theory. ■

3.2 Common-information based sufficient statistic for the transmitter and the receiver

Following Nayyar et al. (2013a), we split the information at the transmitter and the receiver into two parts: *common information* (which is the data that that is known to all future decision makers) and *local information* (which is the total data minus the common information). In particular, at the transmitter the common information is $Y_{0:t-1}$ and the local information is X_t while at the receiver the common information is $Y_{0:t}$ and the local information is empty. Now, consider the following centralized stochastic control problem, which we call the *coordinated system.* At time t, a virtual coordinator observes $Y_{0:t-1}$ (the common