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Efficient Criteria for Stability of Large-Scale Networked Control Systems ⋆

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Abstract: In this paper, we analyze the stochastic stability of a class of large-scale networked control systems. We specifically consider spatially connected linear time-invariant systems whose communication with adjacent subsystems is subject to data losses. We first show that a direct application of the stability theory of Markov jump linear systems provides stability conditions in terms of the eigenvalues of a matrix whose size grows exponentially with the number of subsystems. To overcome this limitation, using the spectrum theory of random matrices, we then derive an alternative stability condition in terms of a matrix whose size grows linearly with the number of subsystems in the network. We illustrate our results with numerical simulations.

Keywords: Networked control systems, Markov jump linear systems, data losses, random matrices

1. INTRODUCTION

Networked control systems have been attracting considerable attention in recent years due to their flexible architectures and less maintenance costs compared with classical wired control systems (Hespanha et al., 2007). The introduction of the notion of networks into control systems, however, also causes technical challenges arising from loss or delay of data and constraints on network capacities. We can find in the literature an enormous advance toward solving these problems, as can be seen in the survey papers (Hespanha et al., 2007; Gupta, 2010).

One of the widely used frameworks for the modeling and analysis of networked control systems is given by a Markov jump linear system (Costa et al., 2013), which is a class of switched linear systems whose switching signal is modeled by a time-homogeneous Markov process. One of the major reasons is that Markov processes provide an efficient model for packet losses and delays that are observed in communications over realistic networks (Smith and Seiler, 2003). Another reason stems from the well-developed theory of Markov jump linear systems, which have enabled the researchers to perform, e.g., output feedback stabilization (Shi et al., 1999), stabilization under random delays in communications (Zhang et al., 2005), and H_∞ control (Seiler and Sengupta, 2005) in the context of networked control systems.

Despite the effectiveness of the framework by Markov jump linear systems, there is a drawback that the framework allows us to study only small scale systems having a few communication components. It was observed by Lee and Bhattacharya (2015) that a simple application of a well-known stability theorem (Fang and Loparo, 2002a) of Markov jump linear systems to a class of large-scale and distributed networked control systems can result in a stability condition in terms of the spectral radius of a matrix whose size depends exponentially on the number of the subsystems. Motivated by this fact, Lee and Bhattacharya (2015) proposed an alternative stability condition in terms of the spectral radius of the eigenvalues of a matrix whose size in general grows much slower than exponential. However, as we observe in this paper, the result implicitly assumes that each connected component in the graph that underlies the networked system is a complete graph. Although Kim et al. (2013) presented a stability condition for a similar class of networked control systems, the systems studied in the paper consists of infinitely many subsystems, whose relevance to realistic networked control systems having finitely many subsystems is not clear.

In this paper, we study stability properties of a class of large-scale, spatially distributed networked control systems. We specifically consider the network of linear time-invariant systems that are connected via unreliable networks modeled by independent time-homogeneous Markov processes. By using the theory of Markov jump linear systems and spectrum theory of large-scale random matrices (Tropp, 2011), we present a sufficient condition for the almost sure stability of the networked control systems. We can efficiently check the stability condition by computing the maximum real eigenvalue of a symmetric matrix whose size grows linearly with respect to the number of the subsystems in the network. It is remarked that, for simplicity of presentation, we confine our attention to the

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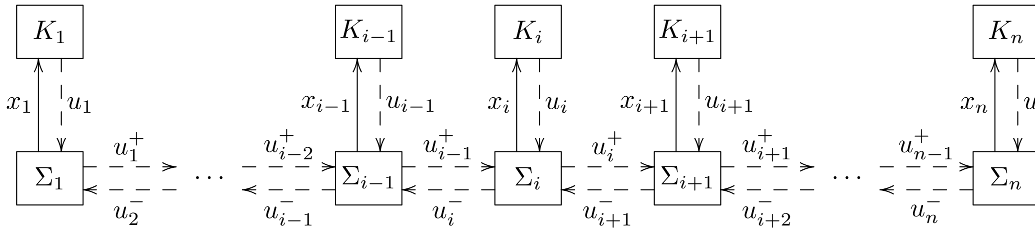

Fig. 1. Networked control system. Arrows represent communication paths without data loss (solid) and with possible data loss (dashed)

case where the subsystems are interconnected via a linear graph as found in vehicle platoons (see, e.g., Jovanović and Bamieh, 2005), although the results obtained in this paper can be generalized to the networked systems having a general topology.

This paper is organized as follows. After giving necessary mathematical notations, in Section 2 we introduce the class of networked control systems studied in this paper. In Section 3, we represent the networked control systems as Markov jump linear systems, and show that application of existing results in the literature yields stability conditions that are not practical. In Section 4, we state computationally efficient stability conditions for the networked control systems. We finally illustrate our results with numerical examples in Section 5.

Mathematical preliminaries For a positive integer n, define the set $[n] = \{1, \dots, n\}$. The Euclidean norm of $x \in \mathbb{R}^n$ is denoted by $\|x\|$. Let I_n denote the $n \times n$ identity matrix. For a matrix A, we denote by $\|A\|$ the maximum singular value of A. If A is square, then we let $\eta(A)$ denote the maximum real part of the eigenvalues of A. When A is symmetric, we denote by $\lambda_{\max}(A)$ the maximum eigenvalue of A. The Kronecker product (resp., Kronecker sum) of matrices A and B is denoted by $A \otimes B$ (resp, $A \oplus B$). Given matrices $A_1, \dots, A_n$, their direct sum, denoted by $\bigoplus_{i=1}^n A_i$, is defined as the block diagonal matrix having the diagonals $A_1, \dots, A_n$. If the matrices $A_1, \dots, A_n$ have the same number of the columns, we denote by $\operatorname{col}(A_1, \dots, A_n)$ the matrix obtained by stacking the matrices $A_1, \dots, A_n$ vertically. For a random matrix X, its expectation is denoted by $E[X]$. If X is symmetric, then the variance of X is defined by $\operatorname{Var}(X) = E[(X - E[X])^2]$. An undirected graph is defined as the pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, n\}$ is a set of nodes and $\mathcal{E}$ is a set of edges consisting of unordered pairs $\{i, j\}$ of distinct nodes $i, j \in \mathcal{V}$. We say that $i, j \in \mathcal{V}$ are adjacent if $\{i, j\} \in \mathcal{E}$. We denote by $\mathcal{N}_i$ the set of nodes that are adjacent to node i.

Let $\theta = \{\theta(t)\}_{t \geq 0}$ be a time-homogeneous Markov process with a finite state space Λ, and let $A(\theta) \in \mathbb{R}^{d \times d}$ for each $\theta \in \Lambda$. A *Markov jump linear system* (Costa et al., 2013) is described by the stochastic differential equation

$$\dot{x}(t) = A(\theta(t))x(t), \tag{1}$$

where $x(0) = x_0 \in \mathbb{R}^{d \times d}$ and $\theta(0) = \theta_0 \in \Lambda$. We say that the Markov jump linear system (1) is *almost surely stable* if there exists $\lambda > 0$ such that $\limsup_{t \to \infty} (\log\|x(t)\|)/t \leq -\lambda$ with probability one for all x_0 and θ_0. The supremum of λ satisfying the above condition is called the decay rate (of almost sure stability). Also, we say that the Markov jump linear system (1) is *mean square stable* if there exist $C \geq 0$ and $\gamma > 0$ such that $E[\|x(t)\|^2]^{1/2} \leq C\|x_0\|e^{-\lambda t}$ for all $t \geq 0$, x_0, and θ_0. The supremum of λ satisfying the above condition is called the decay rate (of mean square stability).

2. PROBLEM FORMULATION

In this section, we present the model of the class of networked control systems studied in this paper. Let n be a positive integer and consider the linear graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}) = ([n], \bigcup_{k=1}^{n-1} \{\{k, k+1\}\})$. We assume that each node of the graph represents the subsystem Σ_i having the continuous-time linear time-invariant dynamics given by

$$\Sigma_i : \dot{x}_i = A_{ii}x_i + B_i u_i + \sum_{j \in \mathcal{N}_i} u_{ij}, \ i \in [n], \tag{2}$$

where $A_{ii} \in \mathbb{R}^{d \times d}$ and $B_i \in \mathbb{R}^{d \times m}$ are constant matrices, $x_i(t) \in \mathbb{R}^d$ is the state of the ith subsystem, $u_i(t) \in \mathbb{R}^m$ is the input from the ith controller to the ith subsystem at time t, and $u_{ij}(t) \in \mathbb{R}^d$ ($j \in \mathcal{N}_i$) denotes the input from the jth subsystem to the ith subsystem at time t. Notice that $\mathcal{N}_i \subset \{i-1, i+1\}$ by the structure of graph $\mathcal{G}$. Based on this spatial structure of the networked system, in the sequel we use the notations $u_{i,i+1} = u_{i+1}^-$ and $u_{i,i-1} = u_{i-1}^+$, whenever $i-1$ or $i+1$ are in the set $[n]$. See Fig. 1 for a schematic picture of the model. We notice that this model contains the case of vehicle platoons (see, e.g., Jovanović and Bamieh, 2005).

We assume that a pair (Σ_i, Σ_{i+1}) ($i = 1, \dots, n-1$) share a common and unreliable communication path, represented by the edge $\{i, i+1\}$, which is not necessarily available at every time instant. For simplicity, in this paper we consider the situation where the control signals sent over a (temporarily) unavailable edge are regarded as being zero by the receivers of the signals. Let us model the availability of the edge $\{i, i+1\}$ by a $\{0, 1\}$-valued stochastic process $\alpha_i = \{\alpha_i(t)\}_{t \geq 0}$ such that $\alpha_i(t) = 1$ if the edge is available at time t and $\alpha_i(t) = 0$ otherwise for each $t \geq 0$. If we assume that the control signals u_i^+ and u_{i+1}^- (if they exist) are given by constant-gain state-feedback, then the input signals u_i^+ and u_{i+1}^- can be expressed as

$$u_i^+(t) = \begin{cases} A_{i+1,i}x_i(t), & \text{if } \alpha_i(t) = 1, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

and

$$u_{i+1}^-(t) = \begin{cases} A_{i,i+1}x_{i+1}(t), & \text{if } \alpha_i(t) = 1, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

for given matrices $A_{i+1,i}, A_{i,i+1} \in \mathbb{R}^{d \times d}$ at each time $t \geq 0$. Similarly, for the communication between the subsystem Σ_i and its local controller, we assume that there exists