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IFAC-PapersOnLine 49-22 (2016) 061–066

On the steady-state behavior of a nonlinear power network model ⋆

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Abstract: In this paper, we consider a dynamic model of a three-phase power system including nonlinear generator dynamics and transmission line dynamics. We derive conditions under which the power system admits a steady-state behavior characterized by an operation of the grid at a synchronous frequency as well as a power balance for each single device. Based on this, we specify a set on which the dynamics of the power grid match the desired steady-state behavior and show that this set is control-invariant if and only if the control inputs to the generators are constant. Moreover, we constructively obtain network balance equations typically encountered in power flow analysis and subsequently show that the power system can be operated at the desired steady-state if and only if the network balance equations can be solved.

Keywords: power system dynamics, steady-state behavior, port-Hamiltonian systems.

1. INTRODUCTION

The electric power system has been paraphrased as the most complex machine engineered by mankind (Kundur, 1994). Aside from numerous interacting control loops, the power system physics are highly nonlinear, large-scale, and contain dynamics on multiple time scales from mechanical and electrical domains. As a result power system analysis and control is typically based on reduced models of various degrees of fidelity (Sauer and Pai, 1998). A widely accepted reduced power system model is a structure-preserving multi-machine model, where each generator model is reduced to the swing equation describing the interaction between the generator rotor and the grid, which is itself modeled at quasi-steady-state via the nonlinear algebraic power balance equations. Despite being based on time-scale separations, quasi-stationarity assumptions, and multiple simplifications, this prototypical model has proved itself useful for power system analysis control (Kundur, 1994; Sauer and Pai, 1998). Nevertheless, the validity of the simplified model has always been a subject of debate; see (Caliskan and Tabuada, 2015; Monshizadeh et al., 2016) for recent discussions.

The modeling, analysis, and control of power systems has seen a surging research activity in the last years. One particular question of interest concerns the analysis of a first-principle nonlinear multi-machine power system model without simplifying generator modeling assumptions and with dynamic (and not quasi-stationary) transmission network models. Fiaz et al. (2013) consider a highly detailed power network model based on port-Hamiltonian system modeling, and they carry out a stability analysis for a single generator connected to a constant linear load. Caliskan and Tabuada (2014) consider a compositional stability analysis of a power network using incremental passivity methods. Their analysis requires, among others, the assumptions of a constant torque and field current at the generators. Unfortunately, their analysis also requires a power preservation property that is hard to verify and whose inherent difficulty is rooted in the $dq0$ coordinates that are convenient for a single generator but incompatible for multiple generators (Caliskan and Tabuada, 2016).

Barabanov et al. (2016) study a single generator in isolation and improve upon the previous papers by requiring milder conditions to certify stability, though it is unclear if the analysis is scalable to a multi-machine system. Related stability analyses have also been carried out also for detailed models of grid-forming power converters that emulate the dynamics of generators (Natarajan and Weiss, 2014; Jouini et al., 2016). Finally, detailed generator models with the grid modeled by quasi-stationary balance equations are studied by Stegink et al. (2016); Dib et al. (2009). In particular, Dib et al. (2009) study existence of equilibria to the nonlinear differential-algebraic model.

In this paper we study the port-Hamiltonian power system model derived from first-principles in abc coordinates by Fiaz et al. (2013). We seek answers to similar questions as in (Dib et al., 2009): under which conditions does there exist a desired steady-state behavior. Our definition of steady-state behavior is inspired by an energy-based framework that suggests that a desired steady-state is characterized by a constant energy in each storage element, and requires that all three-phase AC signals are balanced, sinusoidal, and of the same synchronous frequency.

We provide an algebraic characterization that relates the state variables, control inputs, and a target synchronous frequency such that the dynamics of the power grid coincide with the target steady-state dynamics. Loosely speaking, this set describes a steady-state locus (Isidori

⋆ This research is supported by ETH funds and the SNF Assistant Professor Energy Grant #160573.

10.1016/j.ifacol.2016.10.373

and Byrnes, 2008) provided that it is invariant. We show that this set is control-invariant if and only if the target synchronous frequency is constant and all torque and field current inputs are constant as well, thereby supporting the assumptions in (Caliskan and Tabuada, 2014).

Moreover, we can trace our algebraic feasibility conditions to a simple criterion: the power system (with constant inputs) admits the desired steady-state behavior if and only if the usual network balance (or power flow) equations can be solved. Further results, including a discussion of rotating coordinate frames, can be found in (Groß et al., 2016). We believe that our analysis is a first step towards a stability study of first-principle power system models formulated in stationary coordinate frame (i.e., *abc*).

This paper is organized as follows: In Section 2, we introduce some basic definitions as well as the first-principle nonlinear dynamical model of a power network. We briefly motivate our approach by considering a power balance condition obtained from the Hamiltonian of the power network and specify desired steady-state dynamics. The main result is presented in Section 3. Finally, the paper closes with some conclusions in Section 4.

2. NOTATION AND PROBLEM SETUP

2.1 Notation

We use $\mathbb{R}$ and $\mathbb{N}$, to denote the set of real numbers and integers, and e.g. $\mathbb{R}_{>0}$ to denote the set of positive real numbers. For column vectors $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ we use $(x, y) = [x^\top \ y^\top]^\top \in \mathbb{R}^{n+m}$ to denote a stacked vector, and for vectors or matrices x, y we use $\text{diag}(x, y) = \left[\begin{smallmatrix} x & 0 \\ 0 & y \end{smallmatrix}\right]$. Furthermore, I_n denotes the identity matrix of dimension n, and $\otimes$ denotes the Kronecker product. Matrices of zeros and ones of dimension $n \times m$ are denoted by $\mathbb{0}_{n\times m}$ and $\mathbb{1}_{n\times m}$, and $\mathbb{1}_n$ denotes a column vector of ones of length n. We use $\arctan2(y, x) : \mathbb{R}^2 \to \mathbb{R}_{[0,\pi)}$ to denote the four-quadrant version of the arctangent function with $\arctan2(0, 0) = 0$.

2.2 Dynamical Model of a Power Network

The power network model used in this work consists of n_g generators with index set $\mathbb{G} = \{1, \ldots, n_g\}$, n_v voltage buses with index set $\mathbb{V} = \{1, \ldots, n_v\}$, and n_t transmission lines with index set $\mathbb{T} = \{1, \ldots, n_t\}$. The voltage buses are partitioned into generator buses $\mathbb{V}_g = \{1, \ldots, n_g\}$ and n_l load buses $\mathbb{V}_l = \{n_g + 1, \ldots, n_g + n_l\}$, i.e. $n_v = n_g + n_l$.

The Model used throughout this manuscript is a variant of the port-Hamiltonian model by Fiaz et al. (2013). The reader is referred to Fiaz et al. (2013) and the references therein for a detailed derivation. We will first present our model and then discuss the differences to Fiaz et al. (2013).

The following assumption is required to prove the main result of the manuscript.

Assumption 1. It is assumed that all three-phase electrical components (resistance, inductance, capacitance) have identical values for each phase.

Generators: A generator with index $k \in \mathbb{G}$ is modeled by

$$\dot{\theta}_k = {M_k}^{-1} p_k \tag{1a}$$

$$\dot{p}_k = -D_k {M_k}^{-1} p_k - \tau_{e,k} + \tau_{m,k} \tag{1b}$$

$$\dot{\lambda}_k = -R_k \mathcal{L}_{\theta,k}^{-1} \lambda_k + \begin{bmatrix} C_k^{-1} q_k \\ v_{f,k} \end{bmatrix}, \tag{1c}$$

where $\lambda_k = (\lambda_{\alpha,k}, \lambda_{\beta,k}, \lambda_{f,k}) \in \mathbb{R}^3$ represents the stator and rotor flux linkage, $q_k = (q_{\alpha,k}, q_{\beta,k}) \in \mathbb{R}^2$ are the charges of the generator bus capacitors, $p_k \in \mathbb{R}$ is the momentum of the rotor, and $\theta_k \in \mathbb{R}$ its angular displacement. The generator is actuated by the voltage $v_{f,k} \in \mathbb{R}$ across the excitation winding of the generator and the mechanical torque $\tau_{m,k} \in \mathbb{R}$ applied to the rotor. The electrical torque acting on the rotor is denoted by $\tau_{e,k} = \frac{\partial}{\partial \theta_k}(\frac{1}{2}\lambda_k^\top \mathcal{L}_{\theta,k}^{-1} \lambda_k) \in \mathbb{R}$. The rotor has inertia M_k, damping D_k, the windings have resistance $R_k = \text{diag}(R_{s,k}, r_{f,k}) \in \mathbb{R}^{3\times 3}$, $R_{s,k} = I_2 \otimes r_{s,k}$ and the inductance matrix $\mathcal{L}_{\theta,k} \in \mathbb{R}^{3\times 3}$ is given by

$$\mathcal{L}_{\theta,k} = \begin{bmatrix} L_{ss,k} & \mathcal{R}_{\theta_k} L_{m,k} \\ L_{m,k}^\top \mathcal{R}_{\theta_k}^\top & L_{rr,k} \end{bmatrix}, \tag{2}$$

where $L_{ss,k} = I_2 \otimes l_{ss,k}$, $L_{m,k} = (l_{m,k}, 0) \in \mathbb{R}^2$, and $l_{ss,k} \in \mathbb{R}_{>0}$, $l_{m,k} \in \mathbb{R}_{>0}$ and $L_{rr,k} \in \mathbb{R}_{>0}$. The rotation matrix $\mathcal{R}_{\theta_k} \in \mathbb{R}^{2\times 2}$ is given by:

$$\mathcal{R}_{\theta_k} = \begin{bmatrix} \cos(\theta_k) & -\sin(\theta_k) \\ \sin(\theta_k) & \cos(\theta_k) \end{bmatrix}. \tag{3}$$

Finally, using skew symmetric matrices j and $\jmath$:

$$j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \jmath = \begin{bmatrix} j & \mathbb{0}_{2\times 1} \\ \mathbb{0}_{1\times 2} & 0 \end{bmatrix},$$

it can be verified that the electrical torque is given by:

$$\tau_{e,k} = \frac{1}{2}\lambda_k^\top (\mathcal{L}_{\theta,k}^{-1} \jmath^\top + \jmath \mathcal{L}_{\theta,k}^{-1}) \lambda_k. \tag{4}$$

Furthermore, the stator current $i_{s,k}$ and excitation current $i_{f,k}$ are given by $i_k = (i_{s,k}, i_{f,k}) = \mathcal{L}_{\theta,k}^{-1} \lambda_k \in \mathbb{R}^3$.

Interconnection Graph: Voltage buses are interconnected by a transmission network. The topology of the transmission network is described by the incidence matrix $E \in \mathbb{R}^{n_v \times n_t}$ of its associated graph. In the remainder matrix $\mathcal{E} \in \mathbb{R}^{2n_v \times 2n_t}$, which can be partitioned as follows, is used:

$$\mathcal{E} = E \otimes I_2 = \begin{bmatrix} \mathcal{E}_{q,1} \\ \vdots \\ \mathcal{E}_{q,n_v} \end{bmatrix} = [\mathcal{E}_{T,1} \ \ldots \ \mathcal{E}_{T,n_t}]. \tag{5}$$

Voltage Buses: The dynamics of the n_g voltage buses connected to the generators with index $k \in \mathbb{V}_g$ are given by

$$\dot{q}_k = -G_k C_k^{-1} q_k - [I_2 \ \mathbb{0}_{2\times 1}] \mathcal{L}_{\theta,k}^{-1} \lambda_k - \mathcal{E}_{q,k} L_T^{-1} \lambda_T, \tag{6}$$

with bus capacitance $C_k = I_2 \otimes c_k$, and bus conductance $G_k = I_2 \otimes g_k$. Finally, the transmission lines are described by the line inductance $L_T = \text{diag}(L_{T,1}, \ldots, L_{T,n_t})$, where $L_{T,k} = I_2 \otimes l_{T,k}$, and the vector of transmission line fluxes $\lambda_T = (\lambda_{T,1}, \ldots, \lambda_{T,n_t}) \in \mathbb{R}^{2n_t}$, where $\lambda_{T,k} = (\lambda_{T,\alpha,k}, \lambda_{T,\beta,k}) \in \mathbb{R}^2$ is the flux of the transmission lines $k \in \mathbb{T}$.

Similarly, the dynamics of the load buses are given by:

$$\dot{q}_k = -G_k C_k^{-1} q_k - \mathcal{E}_{q,k} L_T^{-1} \lambda_T, \qquad \forall k \in \mathbb{V}_l, \tag{7}$$

where the conductance G_k models linear resistive loads.