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Dissipativity Reinforcement in Network System: A Design Principle of Large-scale Dynamical System ⋆

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Abstract: We analyze the dissipativity in a large-scale network system, which is composed of a large number of internally connected components. Assuming that every component has special passivity property, we show that the network system inherits the same property of the component independently of the network scale. In addition, the dissipation performance of the network system, evaluated by the $\mathcal{H}_\infty$ norm, is gradually reinforced via network expansion. This analysis can be a fundamental principle of constructing large-scale dynamical systems.

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1. INTRODUCTION

This paper is devoted to analysis of dissipativity and its reinforcement in a large-scale network system.

Realistic network systems are not large-scale at the first stage, but are gradually built up. Let us consider a power system in next generation, which is composed of a large number of natural energy generations such as solar, wind, or thermal power generations. By gradually involving such generations, the entire power system expands its own scale and becomes a large-scale network. We need to design the expanding power network such that the electricity is stably supplied to consumers at any expanding stage. In addition, the network should keep a desired constant power frequency under large fluctuations caused by the natural energy resources.

There have been studies that address system design and control problems of large-scale systems, particularly focusing on their scale-expansion: Additional components are connected to a base system one after another and the scale of the entire system is gradually expanded. See the papers by Tan and Ikeda (1990); Stoustrup (2009); Antsaklis et al. (2013); Goodwine and Antsaklis (2013); Sadamoto et al. (2015). For such expanding systems, problems of stability analysis and stabilization are addressed under the concepts namely expanding construction by Tan and Ikeda (1990), plug and play control by Stoustrup (2009), compositional stabilization by Goodwine and Antsaklis (2013); Antsaklis et al. (2013), and so on. Another approach to stabilization of expanding systems is passivity-based design, see the works by e.g., Moylan and Hill (1978); Bai et al. (2011). The passivity theorem (Zames (1966)) states that assuming that each component is passive, the network system constructed in a specified connection rule is passive. On the basis of the theory, stable expanding systems can be designed.

The strategy for system design in such conventional works does *not impair* the stability or *not deteriorate* the performance of the entire system by expansion. The final goal of this work, exploiting the expansion, we aim to find a design principle where the network expansion *strictly and gradually improves* its performance.

In this paper, we analyze the dissipativity in a large-scale network system, which is composed of a large number of internally connected components. Assuming that every component has special passivity property, we show that the network system inherits the same property of the component independently of the network scale. In addition, the dissipation performance of the network system, evaluated by the $\mathcal{H}_\infty$ norm, is gradually reinforced via network expansion. This analysis can be a fundamental principle of constructing large-scale dynamical systems.

Notation: The symbol $\overline{\sigma}\{\cdot\}$ represents the maximum singular value of a matrix. The symbol $\mathcal{RH}_\infty$ is the set of all proper and complex rational stable transfer functions. For a linear time-invariant system Σ whose transfer functions $\Sigma(s)$ is in $\mathcal{RH}_\infty$, the $\mathcal{H}_\infty$ norm is defined by

$$\|\Sigma\|_{\mathcal{H}_\infty} := \sup_{\mathrm{Re}[s]>0} \overline{\sigma}\{\Sigma(s)\}.$$

The symbol $\mathcal{C}(c, r)$ represents a disk on the complex plane whose center and radius are given by $(c, 0)$ and r, respectively:

$$\mathcal{C}(c, r) := \{x + yi \in \mathbb{C} \mid (x - c)^2 + y^2 \le r^2\}.$$

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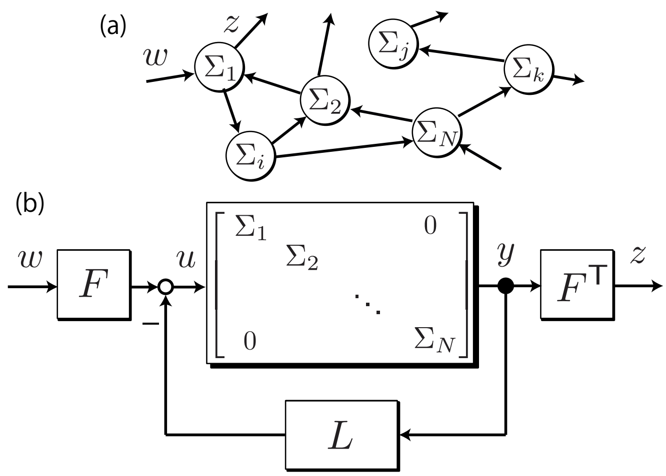

Fig. 1. Network system Σ_{NW}. A number of subsystems Σ_i, $i \in \{1, 2, \ldots, N\}$ are connected each other in a specified rule as in (a). The network system illustrated in (a) is expressed as the feedback form as in (b).

2. SYSTEM DESCRIPTION AND DISSIPATIVITY

In this paper, we consider the network system Σ_{NW} illustrated in Fig. 1. Each subsystem Σ_i, $i \in \{1, 2, \ldots, N\}$ is a linear time-invariant (LTI) system described by the state equation:

$$\Sigma_i : \begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i + D_i u_i, \end{cases} \quad i \in \{1, 2, \ldots, N\},$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^m$, $y_i \in \mathbb{R}^m$ denote the state, input, and output of Σ_i, and $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m}$, $C_i \in \mathbb{R}^{m \times n_i}$, and $D_i \in \mathbb{R}^{m \times m}$ are the constant matrices. Let $w \in \mathbb{R}^m$ be the external input to the entire network Σ_{NW} and $u := [\, u_1^\mathsf{T} \; u_2^\mathsf{T} \; \cdots \; u_N^\mathsf{T} \,]^\mathsf{T}$ and $y := [\, y_1^\mathsf{T} \; y_2^\mathsf{T} \; \cdots \; y_N^\mathsf{T} \,]^\mathsf{T}$. Then, the connection rule in Σ_i, $i \in \{1, 2, \ldots, N\}$ is defined as

$$u = Fw - Ly, \tag{1}$$

where $F \in \mathbb{R}^{mN \times m}$ and $L \in \mathbb{R}^{mN \times mN}$ are the constant matrices, which imply the effect of w to Σ_i and the interconnection between Σ_i, $i \in \{1, 2, \ldots, N\}$, respectively. We assume that F is of full column rank. Further, we define the output of Σ_{NW} by $z \in \mathbb{R}^m$:

$$z = F^\mathsf{T} y, \tag{2}$$

which is utilized for performance evaluation of Σ_{NW}.

Now, we give a definition of dissipativity (Willems (1972)). For a matrix $\Pi \in \mathbb{R}^{2m \times 2m}$, we define a function

$$s(\Pi, w, z) := \begin{bmatrix} w \\ z \end{bmatrix}^\mathsf{T} \Pi \begin{bmatrix} w \\ z \end{bmatrix}. \tag{3}$$

Definition 1. The system Σ_i is said to be dissipative w.r.t. Π if there exists a continuously differentiable and positive semi-definite function $V_i : \mathbb{R}^{n_i} \mapsto \mathbb{R}_+$ such that

$$\dot{V}_i(x_i) := \frac{\partial V_i(x_i)}{\partial x}(A_i x_i + B_i u_i) \le s(\Pi, u_i, y_i) \tag{4}$$

holds for all $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^m$, and $y_i = C_i x_i + D_i u_i$.

In a similar manner, the dissipativity can be defined for Σ_{NW} if it is well-posed. The dissipativity plays an important role for characterizing a class of Σ_i and for evaluating the performance of Σ_{NW}.

Although the dissipativity by Willems (1972) is originally defined with a more general function, that in Definition 1 is with the quadratic function $s(\Pi, u_i, y_i)$. We further restrict Π as

$$\Pi(a, b) := \begin{bmatrix} -a & 1 \\ 1 & -b \end{bmatrix} \otimes I_m,$$

where a and b are non-negative constants. Then, we give the notation.

Notation 2. For given non-negative constants a and b, the symbol $\mathcal{D}(a, b)$ denotes a set of dissipative systems w.r.t. $\Pi(a, b)$.

Remark 3. The definition of $\mathcal{D}(a, b)$ is compared with conventional passive systems. The system Σ is said to be passive, input-strictly passive (ISP), output-strictly passive (OSP), and very-strictly passive (VSP) (see e.g., the work by Hill and Moylan (1976)) if for some positive constants a and b, Σ is in $\mathcal{D}(0, 0)$, $\mathcal{D}(a, 0)$, $\mathcal{D}(0, b)$, and $\mathcal{D}(a, b)$, respectively. Such conventional definitions describe *qualitative* property of a dynamical system. On the other hand, $\mathcal{D}(a, b)$ explicitly describes quantitative performance of a passive system. Quantitative performance is integrated into the passivity in the works by Sakamoto and Suzuki (1996); Oishi (2010); Antsaklis et al. (2013); Zhu et al. (2014) as well. In them, γ-passivity and passivity indices are defined. The class $\mathcal{D}(a, b)$ in this paper is reduced to γ-passive systems or systems with passivity indices by choosing a and b appropriately.

In the following remark and lemma, the class $\mathcal{D}(a, b)$ is interpreted in the frequency domain.

Remark 4. Let a and b be respectively non-negative and positive constants satisfying $ab \le 1$. Further let Σ be a single-input-single-output (SISO) system. Then, the Nyquist plot of $\Sigma \in \mathcal{D}(a, b)$ is included in $\mathcal{C}(c, r) \in \mathbb{C}$, where $c := 1/b$ and $r := c\sqrt{1 - ab}$. Furthermore, we see that the maximum gain of $\Sigma \in \mathcal{D}(a, b)$ is bounded by $c + r$.

Even for a multiple-input-multiple-output system $\Sigma \in \mathcal{D}(a, b)$, the dissipation performance can be interpreted in the frequency domain. See the following lemma.

Lemma 5. Let a and b be respectively non-negative and positive constants satisfying $ab \le 1$. Then, $\Sigma \in \mathcal{D}(a, b)$ satisfies

$$\left(\bar{\Sigma}(j\omega) - \frac{1}{b} I_m\right)^* \left(\bar{\Sigma}(j\omega) - \frac{1}{b} I_m\right) \le \frac{1 - ab}{b^2}, \quad \forall \omega \in \mathbb{R}, \tag{5}$$

where $\bar{\Sigma}(s)$ is the transfer function matrix of Σ. In addition, the $\mathcal{H}_\infty$ norm of Σ satisfies

$$\|\Sigma\|_{\mathcal{H}_\infty} \le \frac{1 + \sqrt{1 - ab}}{b}. \tag{6}$$

Proof. In a similar way to Chapters 2 and 4 in the book by Brogliato et al. (2006), from (4) we have

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