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Optimal Remote Estimation Over Use-Dependent Packet-Drop Channels

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Abstract: Consider a discrete-time remote estimation system formed by an encoder, a transmission policy, a channel, and a remote estimator. The encoder assesses a random process that the remote estimator seeks to estimate based on information sent to it by the encoder via the channel. The channel is affected by Bernoulli drops. The instantaneous probability of a drop is governed by a finite state machine (FSM). The state of the FSM is denoted as the channel state. At each time step, the encoder decides whether to attempt a transmission through the packet-drop link. The sequence of transmission decisions is the input to the FSM. This paper seeks to design an encoder, transmission policy and remote estimator that minimize a finite-horizon mean squared error cost. We present two structural results. The first result in which we assume that the process to be estimated is white and Gaussian, we show that there is an optimal transmission policy governed by a threshold on the estimation error. The second result characterizes optimal symmetric transmission policies for the case when the measured process is the state of a scalar linear time-invariant plant driven by white Gaussian noise. Use-dependent packet-drop channels can be used to quantify the effect of transmission on channel quality when the channel is powered by energy harvesting. In the expanded version of this paper, an additional application to a mixed initiative system in which a human operator performs visual search tasks is presented.

Keywords: State Estimation, Optimal Estimation, Dynamic Channel Assignment, Communication Channel, Energy Management Systems, Channels with Memory

1. INTRODUCTION

Encoders often select varying channel modes to enhance transmission performance in the presence of power and energy constraints. For example, in battery-operated wireless communication systems with energy harvesting, the decision of whether to attempt transmission must be made time and again at each time-step. The charge-level of the battery induces memory in the channel. We define a class of *use-dependent packet-drop channels* to model the effect of attempted transmissions on current and future performance, which in our case is quantified by the probability that an attempted transmission is dropped. The memory in use-dependent packet-drop channels is modeled by a finite state machine (FSM). The state of the FSM, or channel state, determines the instantaneous probability of drop. In our formulation the input to the FSM is the time-sequence of decisions of whether to attempt a transmission.

We consider a system formed by a remote estimator, a transmission policy, a use-dependent packet-drop channel and an encoder. The estimator produces an estimate of the state of a linear time-invariant plant that is accessible to the encoder. The estimate is based on information transmitted from the encoder to the estimator via the channel. The encoder and transmission policy also have access to past transmission decisions and channel feedback on the realization of current and past drops. The encoder determines what to transmit over the channel and the transmission policy determines when to attempt a transmission. The main goal of this paper is to investigate encoders, transmission policies and remote estimators that jointly minimize the mean squared state estimation error over a finite time-horizon. Section 2 contains the problem formulation.

1.1 Outline of the main results

The following are our two main results characterizing the structure of optimal transmission policies for our problem. In both results, no restrictions are placed on the dynamics or size of the FSM.

In the first result, we assume that the process to be estimated is white and Gaussian. We show that the optimal transmission policy is of the threshold type, meaning that the encoder chooses to attempt transmission when the process takes values outside a certain interval $[\underline{\tau}, \bar{\tau}]$. The

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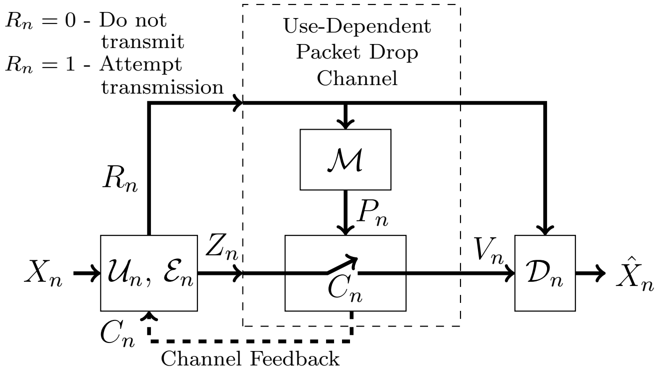

Fig. 1. The problem under investigation is a remote estimation problem over a packet-drop channel, whose probability of drop P_n is governed by the Finite State Machine $\mathcal{M}$.

characteristics of the use-dependent packet-drop channel determine the values of $\bar{\tau}$ and $\underline{\tau}$. In general, $\bar{\tau}$ may not equal $-\underline{\tau}$, even when the process is zero-mean.

In the second result, the process to be estimated is the state of a scalar linear time-invariant plant driven by white Gaussian noise, for which we seek to obtain an optimal symmetric transmission policy. We show that if the channel performs satisfactorily in all channel states, then there exists at least one symmetric threshold that, when applied to the estimation error, leads to a transmission policy that is optimal among all symmetric strategies.

In section 2, the formal definition of use-dependent packet-drop channels is given and the problem is formulated. Section 3 presents the technical results. Section 4 outlines an application of our results to an energy harvesting channel.

An expanded version of this paper is available on arXiv, Ward and Martins (2016). The expanded version includes the proofs of Theorem 2 and Lemma 7, an additional application to a mixed initiative system in which a human operator performs visual search tasks, and an appendix presenting basic concepts on quasi-convex functions. In the mixed initiative application, a packet-drop models the human operator ignoring a visual search task request. The probability of the operator ignoring a request depends on the operator's workload and bias which are modeled by a specific FSM.

1.2 Related Literature

In Lipsa and Martins (2009) and Lipsa and Martins (2011), an estimation problem over a packet drop channel with communication costs is considered. In contrast to Lipsa and Martins (2009) and Lipsa and Martins (2011), here we introduce a channel state and do not consider explicit communication costs in the objective function. In our formulation, the channel state, which depends on current and past transmission decisions, and its impact on channel performance create an implicit communication cost. For example, in the energy harvesting application discussed in section 4, there is no explicit cost for attempting a transmission. However, attempting a transmission reduces the energy available for future transmissions, which causes performance degradation that can be viewed as an implicit cost for attempting a transmission.

Considering costly measurements (or transmissions) in estimation and control problems has a long history and has been modeled in many ways. In Athans (1972), one of several possible measurements with different observation costs is selected to minimize a combination of error and observation cost. In Shamaiah et al. (2010), a subset of the measurements is selected in order to minimize the log-determinant of the error covariance. In Sinopoli et al. (2004), the arrival of observations is a random process and the convergence of the error covariance is studied. In Hajek et al. (2008), the task is to locate a mobile agent and the observation cost is the expected number of observations that must be made to do so.

In Weissman (2010), the capacity of channels with action-dependent states is studied. Although our problem formulation is similar to that of Weissman (2010) in motivation, it differs in several accounts. In contrast to Weissman (2010), we consider finite time horizons, a mean-squared error cost and a new class of packet-drop channels.

2. PROBLEM FORMULATION

2.1 Notation

We use calligraphic font ($\mathcal{F}$) to denote deterministic functions, capital letters (X) to represent random variables and lower case letters (x) to represent realizations of the random variables. Let $\mathcal{N}(0, \sigma^2)$ denote the Gaussian distribution with zero mean and variance σ^2. We use Q^n to denote the finite sequence $\{Q_1, Q_2, \dots Q_n\}$. The real line is denoted with $\mathbb{R}$ and a subset of $\mathbb{R}$ is denoted with double barred font, such as $\mathbb{A}$. The indicator function of a set $\mathbb{A}$ is defined as

$$\mathbf{1}_{\mathbb{A}}(x) \overset{def}{=} \begin{cases} 1 & x \in \mathbb{A} \\ 0 & \text{Otherwise.} \end{cases}$$

The expectation operator is denoted with $E[\cdot]$. By $\lim_{\delta \downarrow 0} \mathcal{F}(\delta)$ we mean the limit of $\mathcal{F}(x)$ at 0 from the right.

2.2 Problem Formulation

Consider the following scalar linear time-invariant system

$$X_{n+1} = aX_n + W_n, \quad n \geq 0, \quad X_0 = x_0,$$

where X_n is the state, a is a real constant, W_n is independent and identically distributed Gaussian noise with zero mean and variance σ^2. The initial state $x_0 \in \mathbb{R}$ is known.

Observations are made by the encoder and transmitted to the remote estimator over a use-dependent packet-drop channel, which is defined below. In figure 1, the dotted box represents the use-dependent packet-drop channel.

Definition 1. (Use-dependent packet-drop channels). Let $\mathcal{M}^s : \mathbb{Q} \times \{0, 1\} \to \mathbb{Q}$ and $\mathcal{M}^o : \mathbb{Q} \to [0, 1]$ be given, where $\mathbb{Q} = \{1, \dots, m\}$ represents the state space for the finite state machine (FSM). The channel inputs are Z_n and R_n, which take values in $\mathbb{R}$ and $\{0, 1\}$, respectively. In this model Z_n represents the information to be transmitted, while the decision to attempt a transmission (or not) is