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Optimal Estimation Algorithm Design under Event-based Sensor Data Scheduling ⋆

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Abstract: This paper investigates a remote state estimation problem over a communication channel, which is of theoretical and practical significance in the study of cyber-physical systems (CPS). In consideration of both bandwidth limitation and estimation quality, an event-based schedule is proposed to decide whether to transmit the current measurement to the remote estimator or not. To overcome the difficulties induced by the bilaterally hidden-truncation process from event-based scheme, we introduce the generalized closed skew normal distribution to accurately portray the state distribution. Furthermore, an exact MMSE estimation algorithm is designed, which does not rely on the Gaussian approximation assumption. Numerical examples are provided to show the effectiveness of the proposed algorithm.

Keywords: CPS, remote state estimation, event-based scheduling, MMSE estimation

1. INTRODUCTION

With the integration of communication, computation and control elements, cyber-physical systems (CPS) enable a wealth of new applications, including environment monitoring and control, smart home, unmanned aerial vehicle and modern battlefields (Chen et al., 2011). Despite of the exciting application prospect, some technical issues must be fully analyzed before CPS become commonplace (Johansson et al., 2014). Among them, much effort has been devoted to examining how to improve estimation performance, with the consideration of limited sensing and communication resources, which motivates the research on the event-based scheduling (Heemels et al., 2012; Han et al., 2015; Cheng et al., 2016).

In many studies, measurement innovation plays the dominating role in the design of event-based schemes. In estimation theory, measurement innovation is defined as the difference between the measurement collected via the sensor of current time, and the one-step measurement prediction computed at the remote estimator of the last step. Innovation reveals the potential contribution of the newly obtained measurement for minimizing the estimation error covariance. In a type of widely used event-based schedule, the packet is sent only when its innovation exceeds the predesignated threshold; Otherwise not since small innovation implies that the measurement prediction at the remote estimator side has already been closed to the real data (Wu et al., 2013; You and Xie, 2013). The prior state distribution under innovation-based scheduling is usually assumed to be Gaussian, which benefits of obtaining the closed-form expression of the estimation error covariance when the packet is kept saving due to its small innovation.

In a slightly different problem setting, Sukhavasi and Hassibi (2013) have indicated that the hidden-truncation process caused by quantization is generally non-Gaussian. Such a finding recalls us the remark of Åström and Bernhardsson (2002): "An interesting conclusion is that the steady state probability distribution of the control error (for event-based sample) is non-Gaussian even if the disturbances are Gaussian." Instead of Gaussian, Sukhavasi and Hassibi (2013) showed that such a process can be properly portrayed via the *Generalized Closed Skew Normal* (GCSN) distribution (Genton, 2004).

In our view, there exist some inherent connections between the quantization problem and event-based scheduling, due to the fact that both lead to the hidden-truncation process. Such a connection motivates us to recheck the Gaussian approximation assumption in the event-based scheme and develop a novel estimation algorithm via GCSN distribution. The main contribution of this paper can be stated as follows. By introducing the GCSN distribution into the event-based scheduling, an exact MMSE estimation algorithm which does not resort to the Gaussian approximation assumption is proposed for the first time for the multidimensional linear Gaussian Markov process.

Notations: $\mathbb{R}^n$ is the n-dimensional Euclidean space. $y_a^b \triangleq \mathrm{col}\{y_a, \ldots, y_b\}$ represents the column vector with entries $y_a, \ldots, y_b$. x' denotes the transpose of a vector or a matrix. $\mathrm{Cov}(x, y) \triangleq \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])']$ is the covariance of random variable ($r.v.$) x and y; $\mathrm{Cov}(x, x)$ is abbreviated as $\mathrm{Var}(x)$. Similarly define the conditional variance $\mathrm{Var}(x|y)$. $p(x)$ represents the probability density function (pdf) of

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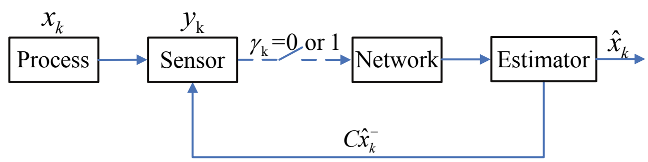

Fig. 1. Event-based sensor data scheduling

a $r.v.$ $\mathbb{P}(a \le x \le b)$ denotes the probability with the limited range $[a, b]$. $\mathcal{N}_n(\mu, \Sigma)$ is the n-dimensional Gaussian distribution with mean μ and variance Σ, corresponding pdf is denoted as $\phi_n(x; \mu, \Sigma)$. $\Phi_n([a, b]; \mu, \Sigma)$ represents the integration of $\phi_n(x; \mu, \Sigma)$ within the limited range $[a, b]$, where, $a, b \in \mathbb{R}^n$ and the integration is component-wise.

2. PROBLEM SETUP

We consider the sensor data scheduling for remote state estimation problem as depicted in Fig. 1. The discrete linear time-invariant process is described as follows:

$$x_{k+1} = Ax_k + \omega_k, \tag{1}$$
$$y_k = Cx_k + v_k, \tag{2}$$

where, $x_k \in \mathbb{R}^n$ is the system state, $y_k \in \mathbb{R}^m$ is the measurement collected via local sensor at time k. Gaussian white noises ω_k and v_k are assumed to be zero-mean with variances $Q \ge 0$ and $R > 0$, respectively. The initial state x_0 is also Gaussian with distribution $\mathcal{N}_n(\mu_0, \Sigma_0)$. Two sequences $\{\omega_k\}$ and $\{v_k\}$ are independent, which implies that $\mathbb{E}[\omega_i v_j'] = 0, (\forall i, j)$. The pair (A, C) is detectable and $(A, \sqrt{Q})$ is stabilizable.

To save scarce communication resource which may be used by other applications, the scheduler equipped at the sensor side decides whether to transmit the current measurement y_k over the network to the remote estimator or not at each time step. Denote $\gamma_k = 1$ as the packet is sent and $\gamma_k = 0$ otherwise, which will be shortly specified in Eq.(5). To concentrate on analyzing the senor scheduling scheme, we ignore other imperfect effects such as packet loss, time delay and quantization, etc. Denote the information set available at the estimator side of time $k \in \mathbb{Z}^+$ as

$$I_k \triangleq \{\gamma_1 y_1, \ldots, \gamma_k y_k\}. \tag{3}$$

Assume that there is a feedback channel from the remote estimator to the local scheduler for reliably transmitting the measurement predict $\hat{y}_k^- \triangleq \mathbb{E}[y_k|I_{k-1}]$ at any time instant $k \in \mathbb{Z}^+$, where, $I_0 \triangleq \{x_0\}$.[1] On that basis, the scheduler can compute the measurement innovation z_k specified as Eq. (4) for indicating how accurately the remote estimator can predict the current measurement y_k before it sends, and thus how to transmit y_k may help to reduce such a prediction error covariance. The measurement innovation is defined as

$$z_k \triangleq y_k - \hat{y}_k^- = y_k - C\hat{x}_k^-. \tag{4}$$

We have $\mathbb{E}[z_k] = 0$ due to projection law (Kailath et al., 2000).

The scheduling sequence $\{\gamma_k\}$ is specified via the following event-based scheme for a given threshold $\delta \in \mathbb{R}^m$:

$$\gamma_k = \begin{cases} 0, & \text{if } -\delta \le z_k \le \delta, \\ 1, & \text{otherwise.} \end{cases} \tag{5}$$

[1] Such a feedback is based on the IEEE 802.15.4/ZigBee protocol. Details can reference Wu et al. (2013).

Here, the inequality holds for the component-wise sense. In such a scheme, saving one step of transmission ($\gamma_k = 0$) does not mean the corresponding error covariance increases as that of open-loop prediction. On the contrary, such an event implies that the measurement predict has already been quite reliable, which helps to reduce the communication cost while keeping the estimation error covariance under a pre-designed level.

The primary concern of this paper is to propose an exact MMSE estimation algorithm for system (1) $\sim$ (2) under scheduling scheme (5). We assume that no a prior probability information of state predict is available.

3. PRELIMINARIES

This paper chooses the Bayesian method as depicted in Ho and Lee (1964) to obtain the MMSE estimation algorithm. To ease analysis, the equalities frequently used in subsequent sections are provided in this section.

1) Compute the prior pdf via Chapman-Kolmogorov equation

$$p(x_{k+1}|I_k) = \int p(x_{k+1}|x_k)p(x_k|I_k)\mathrm{d}x_k. \tag{6}$$

2) Obtain the posterior pdf via Bayesian rule on account of the event really happens:
   - In the case of $\gamma_{k+1} = 1$, i.e., $I_{k+1} = I_k \cup \{y_{k+1}\}$.

$$p(x_{k+1}|I_{k+1}) = \frac{p(y_{k+1}|I_k, x_{k+1})p(x_{k+1}|I_k)}{p(y_{k+1}|I_k)}. \tag{7}$$

   where,

$$p(y_{k+1}|I_k) = \int p(y_{k+1}|I_k, x_{k+1})p(x_{k+1}|I_k)\mathrm{d}x_{k+1}. \tag{8}$$

   - In the case of $\gamma_{k+1} = 0$, i.e., $I_{k+1} = I_k \cup \{-\delta \le z_{k+1} \le \delta\}$.

$$p(x_{k+1}|I_{k+1}) = p(x_{k+1}|I_k)\frac{\mathbb{P}(-\delta \le z_{k+1} \le \delta|I_k, x_{k+1})}{\mathbb{P}(-\delta \le z_{k+1} \le \delta|I_k)}. \tag{9}$$

Different from classic Kalman filter, the integrations in (6) and (8) are computationally prohibitive, since γ_k cannot be expressed as the linear combination of the elements of I_k, which clearly reveals the inherent nonlinear property of the event-based mechanism. For facilitate of analysis, Wu et al. (2013); You and Xie (2013) assumed that the prior state distribution is Gaussian, i.e.,

$$p(x_{k+1}|I_k) = \phi(x_{k+1}; \hat{x}_{k+1}^-, P_{k+1}^-), \tag{10}$$

where, $\hat{x}_{k+1}^- \triangleq \mathbb{E}[x_{k+1}|I_k]$, $P_{k+1}^- \triangleq \mathbb{E}[(x_{k+1} - \hat{x}_{k+1}^-)(\cdot)']$. Such an assumption substantially simplifies the computation and approximates the corresponding filter to be linear. Though nice estimation algorithm is obtained, the rationality of such a Gaussianality approximation lacks rigorous verification.

In next two sections, the Gaussian approximation assumption is shown to break down and an exact MMSE estimation algorithm, which does not resort to any approximation, is also provided.