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Stability analysis of mobile robot formations based on synchronization of coupled oscillators

Tadashi Nakamura * **Miyuki Tsukiji** * **Naoyuki Hara** * **Keiji Konishi** *

* *Department of Electrical and Information Systems, Osaka Prefecture University, 1–1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531 Japan (e-mail: n-hara@eis.osakafu-u.ac.jp)*

Abstract: The present paper deals with mobile robot circular formations realized by a simple control law. This law is established by utilizing synchronization of coupled limit-cycle oscillators. The stability of formations is analyzed, and the analytical results give a simple procedure for designing a control law. This procedure allows us to choose a desired circular formation. Furthermore, the analytical results guarantee that we can specify any radius of circular formations. The results are verified by some numerical examples.

Keywords: Stability, Mobile robot, Formation control, Coupled oscillators, Synchronization

1. INTRODUCTION

Unmanned vehicles have been used for a wide range of purposes such as planetary exploration, environmental monitoring and investigation/restoration work in dangerous places (e.g., damaged nuclear plants). In recent years, the cooperative use of multiple vehicles has attracted interest in both academia and industry [Kumar et al. (2005); Mesbahi and Egerstedt (2010)]. Formation control is such an example where multiple vehicles are required to form a specific formation pattern. Motivations for achieving formations come from various reasons, depending on a specific application; e.g., to realize an optical interferometer array by making satellites, each of them is assumed to be equipped with a telescope, form a desired spatial pattern [Mesbahi and Egerstedt (2010)]. In formation control, of particular interest is the case when each vehicle makes a decision by itself with only local measurements/communications, and a global formation pattern is achieved as a result of the decisions made by each vehicle, i.e., there is no centralized controller that supervises all the vehicles.

In our previous study [Yamada et al. (2011)], we focused on circular formations for multiple mobile robots based on a control law we developed for a single robot [Hara et al. (2010)]. Circular formations of mobile robots have some possible applications for enclosing/capturing a target object, and collecting environmental data [Dunbabin and Marques (2012)]. The control law [Hara et al. (2010)] was designed using a nonlinear system with limit cycle, and a formation control law was obtained based on this result. Although this method was demonstrated by numerical simulation and experiment, the approach adopted there does not allow the in-depth theoretical analysis of the formation control law.

Complex network science has created considerable interest in various fields [Strogatz (2003)]. It is well accepted that coupled oscillators have played a key role in complex network science [Pikovsky et al. (2001)]. A number of studies have been made on the collective phenomena in coupled oscillators in order to clarify the fundamental mechanism of various types of synchronization appeared in complex networks [Boccaletti et al. (2006); Arenas et al. (2008)]. In recent years, the mechanism is widely used in engineering applications, such as locomotion control of a biomimetic underwater vehicle [Zhou and Low (2012)], synchronization of pulse-coupled oscillators for wireless sensor networks [Okuda et al. (2011)], peak power reduction in energy storage oscillators coupled by delayed power price [Fukunaga et al. (2016)], and so on [In et al. (2009)].

Our previous study suggested that synchronization in coupled oscillators, which has been intensively investigated in the field of nonlinear physics, can be used for control of mobile robot circular formations [Hara et al. (2013)]. This suggestion was experimentally verified with two-wheeled mobile robots [Tsukiji et al. (2014)]. Unfortunately, these our previous studies have major drawbacks:

(i) It is difficult to analyze the stability of formations;
(ii) The radius of circular formations cannot be specified.

The drawback (i) represents that we cannot design parameters for desired formations. The drawback (ii) implies that all robots run on a common circle path; in other words, we cannot avoid collision of robots.

The main purpose of the present paper is to overcome the drawbacks (i) (ii). We show that the key idea for overcoming them is to use only the polar coordinates for stability analysis, while our previous studies used both

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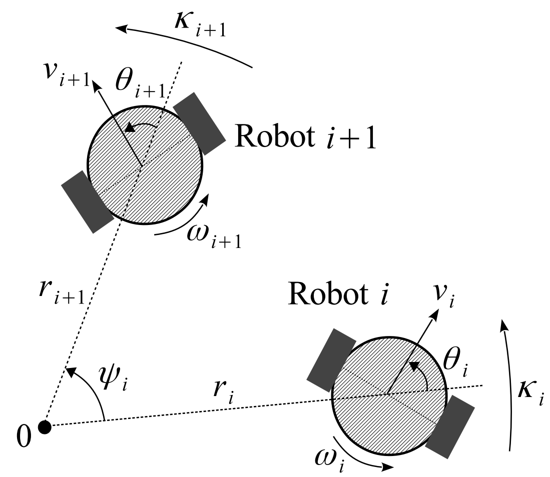

Fig. 1. Two-wheeled mobile robots.

polar and rectangular coordinates. This idea drastically simplifies the stability analysis; accordingly, we can provide a procedure for designing the parameters, and can specify the radius. These analytical results are verified on some numerical simulations.

2. CONTROL LAW FOR CIRCULAR FORMATIONS

Let us consider two-wheeled mobile robots sketched in Fig. 1. We assume the kinematic model for the robots, and the dynamics of robot $i \in \{1, \dots, N\}$ is described by

$$\begin{bmatrix} \dot{r}_i \\ r_i\kappa_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \tag{1}$$

where $r_i > 0$ denotes the distance between the robot i and the origin. $\kappa_i \in \mathbb{R}$ and $\theta_i \in \mathbb{R}$ are the angular velocity of the robot i around the origin and the angle between the heading of the robot i and its radial direction, respectively. The angle between the radial direction of the robot i and that of $i+1$ is denoted by $\psi_i \in \mathbb{R}$.

The robot i is controlled by the following inputs: the heading-direction component of velocity ($v_i \in \mathbb{R}$) and the angular velocity around the center of robot i ($\omega_i \in \mathbb{R}$). These control inputs are given by driving the two wheels with appropriate rotational velocities. This paper proposes a control law of these inputs which induce circular formations. It should be emphasized that our previous studies [Hara et al. (2013); Tsukiji et al. (2014)] employed both of the polar coordinates and rectangular coordinates to express the dynamics of the robots, while the present paper employs only the polar coordinates.

Now we turn our attention to the reference dynamics of radical direction and phases,

$$\dot{r}_i = f(r_i, \hat{r}_i) := a r_i \left(1 - \frac{{r_i}^2}{\hat{r}_i^2}\right), \tag{2a}$$

$$\kappa_i = g(\psi_i) := \Omega + \varepsilon \sin\psi_i. \tag{2b}$$

This paper utilizes these reference dynamics to establish the control law. Note that the dynamics (2a) depends only on the i-th distance r_i. On the other hand, the dynamics (2b) depends on the i-th and $(i+1)$-th phases, and is one of the simplest models of well-known coupled phase oscillators [Acebrón et al. (2005); Scardovi et al. (2007)].

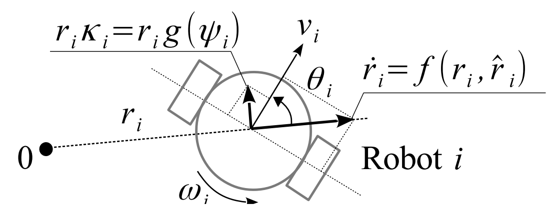

Fig. 2. Illustration of vector field of dynamics of (2a), (2b) and robot i.

Remark that ψ_N denotes the angle between the radial direction of robot 1 and that of N; thus, the dynamics (2b) represents one-way *ring* coupled phase oscillators. $\hat{r}_i > 0$ denotes the stable equilibrium point of the dynamics (2a). The parameters, $a \in \mathbb{R}$ and $\Omega \in \mathbb{R}$, express the convergence rate for $\hat{r}_i$ and the angular velocity around the origin. $\varepsilon \in \mathbb{R}$ is the coupling strength.

We establish the control law for v_i and ω_i such that the two-wheeled mobile robots (1) behave in the reference dynamics. In Fig. 2, the vector field of the reference is written over the robot i. The reference velocities in radial direction (i.e., $\dot{r}_i$) and in its vertical direction (i.e., $r_i\kappa_i$) are given by the reference dynamics (2). This paper proposes the following control law: the velocity in heading direction v_i and the angular velocity ω_i are respectively set to be proportional to the reference velocities in the radial direction and in the vertical direction,

$$v_i = \overline{v}_i := k_v \left\{ f(r_i, \hat{r}_i)\cos\theta_i + r_i g(\psi_i)\sin\theta_i \right\}, \tag{3a}$$

$$\omega_i = \overline{\omega}_i := k_\omega \left\{ r_i g(\psi_i)\cos\theta_i - f(r_i, \hat{r}_i)\sin\theta_i \right\}, \tag{3b}$$

where $k_v, k_\omega \in \mathbb{R}$ are feedback gains. In real situations, it is difficult to realize large velocities v_i and ω_i due to practical limitation of the wheel rotational velocity. For such cases, k_v and k_ω have to be small to reduce the velocities.

Now let us restrict our attention to a situation where the dynamics of robot i, which is described by Eq. (1), is controlled by the law (3). In order for the robot i to be autonomously controlled, the robot i has to get the following three real-time measurements used in the control law (3) (see Fig. 1).

(a) r_i: distance to the origin.
(b) θ_i: angle to the heading direction.
(c) ψ_i: angle to the robot $i+1$.

While the requirement of these measurements comes from the form of the control law we adopted, it is also feasible in practice. This paper supposes that every robot has sensors for measuring the distance to the origin r_i, distance to the next robot $i+1$, the angle between its radial and the heading directions, and the angle between its radial and $(i+1)$-th robot directions. Each robot gets the measurements (a) and (b), but cannot (c) directly. It is easy to see that the cosine formula allows the robot i to compute (c) in real-time by using the measured data, r_i, the distance to the next robot $i+1$, and the angle between its own radial direction and the direction to the robot $i+1$. In consequence, our robots can be considered as fully autonomous robots.