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IFAC-PapersOnLine 49-22 (2016) 198–203

Multi-robot Formation Control over Distance Sensor Network

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Abstract: This paper proposes a formation control method by using one-dimensional (1D) distance sensors, e.g. LED-based PSD sensors. These sensors enhance the mobility of the multi-robots because they can be equipped with small-payload robots. Note that only by the information on 1D distances from other robots, their center positions cannot be correctly identified. Moreover, the robots have to control their attitudes to detect other robots, which is a special requirement for using the 1D distance sensors. In this paper, we design a formation controller which requires only the information on the 1D distances. The controller consists of two parts: the position control to achieve formation and the attitude control to direct the sensors to other robots. Moreover, we propose a sensor detection network topology with which the formation is successfully achieved. Finally, the effectiveness of the proposed method is illustrated by a simulation result.

Keywords: Formation control, multi-robots, distributed control, LED-based PSD distance sensors, sensor detection network

1. INTRODUCTION

Recently, formation control has been vigorously investigated in the field of the control engineering, e.g. Fax and Murray (2004); Anderson et al. (2008); Krick et al. (2009); Oh et al. (2015). In the existing researches, they design distributed controllers which use the measurement of the relative positions (i.e. distances and directions) of nearby robots. To measure the relative positions, cameras or laser rangefinders are usually used (Vidal et al. (2003); Renaud et al. (2004); Vig and Adams (2006)). On the other hand, we focus on *one-dimensional (1D) distance sensors* such as LED(light-emitting diode)-based PSD (position sensitive detector) sensors (Siegwart et al. (2004); Pastorius (2001)). Thse sensors are much cheaper and easier to deploy, and are suitable to small-payload robots.

However, each 1D distance sensor can measure toward only one fixed direction angle. We assume that the detection angles cannot be changed by actuators, e.g. servo motors, because of the simplicity of robots. Then, each robot has to maintain its attitude to direct the sensors toward other robots. Moreover, only by the information on 1D distances from other robots, we cannot correctly identify the center positions of the robots. In this way, formation control by using 1D distance sensors is a challenging problem compared with the existing researches which assume that relative positions are obtainable regardless of the attitudes of the robots.

In this paper, we show that formation control is achievable by using 1D distance sensors under an appropriate sensor network topology. First, we design a controller based on the gradient-flow approach (Biyik and Arcak (2007); Zhu and Martinez (2013); K. Sakurama, S. Azuma and T. Sugie (2015)). Then, we convert the derived controller into a *distributed* controller in the sense that each robot requires only its own sensor information. This controller consists of two parts: the position control to achieve formation and the attitude control to direct the sensors to the others. The attitude control is a special requirement for using 1D distance sensors. Moreover, we propose a sensor network topology with which the formation is successfully achieved. The effectiveness of the proposed method is illustrated by a numerical example.

In our setting, the robots have to control not only their positions, but also their attitudes to keep detection by the 1D distance sensors. This is difference from the exiting studies on distance-based formation (Dimarogonas and Johansson (2009); Oh and Ahn (2011)) because they assume that the distances from neighbors can be detected in any attitudes. This can be possible if robots are equipped with cameras. However, in this paper, the cheaper and easier sensors, namely the 1D distance sensors, are employed.

Notations: Let $\mathbb{R}$ be the set of the real numbers. For a vector $x \in \mathbb{R}^2$, the Euclidean norm is given as $\|x\| = \sqrt{x^\top x}$. For non-zero vectors $x, y \in \mathbb{R}^2$, $\angle(x, y) \in (-\pi, \pi]$ represents the signed angle from x to y such that

$$\sin(\angle(x,y)) = \frac{\det([x\ y])}{\|x\|\|y\|},\ \cos(\angle(x,y)) = \frac{x^\top y}{\|x\|\|y\|},$$

where $\det(\cdot)$ is the determinant of a matrix. The rotation matrix $R : (-\pi, \pi] \to \mathbb{R}^{2\times 2}$ of the angle $\theta \in (-\pi, \pi]$ is described as

* This work was partly supported by JSPS KAKENHI Grant Number 15K06143.

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.396

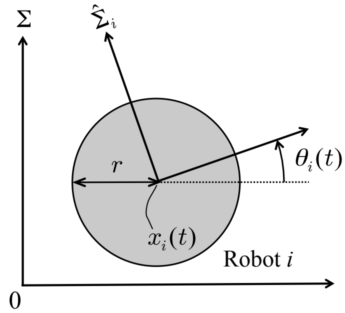

Fig. 1. World coordinate system Σ and relative coordinate system $\hat{\Sigma}_i$

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}. \quad (1)$$

The unit vector $e : (-\pi, \pi] \to \mathbb{R}$ toward the angle $\theta \in (-\pi, \pi]$ is described as

$$e(\theta) = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}. \quad (2)$$

For a finite countable set $\mathcal{N}$, let $|\mathcal{N}|$ be the number of its elements. For functions $f_i(x)$, $i = 1, 2, \ldots, n$ and an index set $\mathcal{N} \subset \{1, 2, \ldots, n\}$, $[f_i(x)]_{i\in\mathcal{N}}$ is the column of the functions $f_i(x)$ corresponding to the elements of $\mathcal{N}$, i.e.

$$[f_i(x)]_{i\in\mathcal{N}} = [f_{i_1}(x)^\top \ f_{i_2}(x)^\top \ \cdots \ f_{i_m}(x)^\top]^\top, \quad (3)$$

where $i_1, i_2, \ldots, i_m \in \mathcal{N}$ and $m = |\mathcal{N}|$.

2. PROBLEM FORMULATION

2.1 Setting of robots

Consider n robots in 2-dimensional space. Assume that the shapes of the robots are given by circles with radius $r > 0$. The set of the numbers of the robots is described as $\mathcal{V} = \{1, 2, \ldots, n\}$. Robot 1 is the *leader* of the group of the robots, and is not affected by the others. The rest of robots $i \in \mathcal{V}\backslash\{1\}$ are called *followers*.

Let Σ be the world coordinate system, where the coordinate of robot $i \in \mathcal{V}$ is represented by $x_i(t) \in \mathbb{R}^2$ at time $t \geq 0$. Let Σ_i be the relative coordinate system of robot i with the origin at the center of robot i and the basis $\{e(\theta_i(t)), e(\theta_i(t)+\pi/2)\}$, where $\theta_i(t) \in (-\pi, \pi]$ is the attitude angle of robot i. See Fig. 1 for the illustration. Assume that each robot is omni-directionally movable with the velocity input $\hat{v}_i(t) \in \mathbb{R}^2$ in $\hat{\Sigma}_i$ and the angular velocity input $\omega_i(t) \in \mathbb{R}$ for robot $i \in \mathcal{V}$. Then, the velocity in Σ and the angular velocity of robot i are described as

$$\dot{x}_i(t) = R(\theta_i(t))\hat{v}_i(t) \quad (4)$$

$$\dot{\theta}_i(t) = \omega_i(t), \quad (5)$$

which represent the state equation of robot i.

Assume that followers are equipped with 1D distance sensors of the same number. Let m be the number of the sensors on each follower. The set of the sensors of robot $i \in \mathcal{V}\backslash\{1\}$ is denoted as $\mathcal{S}_i = \{1, 2, \ldots, m\}$. The sensor $s \in \mathcal{S}_i$ is put on robot i at the coordinate $\hat{p}_{is} \in \mathbb{R}^2$ in

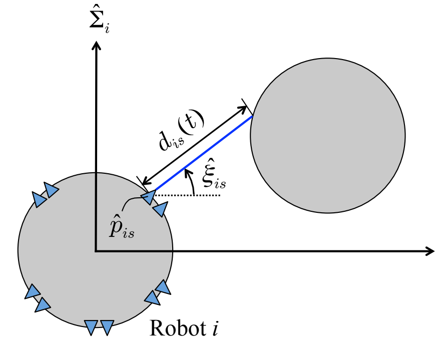

Fig. 2. Arrangement of 1D distance sensors

$\hat{\Sigma}_i$ toward the detection angle $\hat{\xi}_{is} \in (-\pi, \pi]$ from the x-axis of $\hat{\Sigma}_i$. See Fig. 2 for the illustration. From sensor s, the signal $d_{is}(t) \geq 0$ is obtained, which represents the distance from the surface of another robot. It should be noticed that only by using $d_{is}(t)$, the center of another robot cannot be correctly identified as shown in Fig. 2. The outputs available to robot i are $d_{is}(t)$ for $s \in \mathcal{S}_i$. Thus, the controllers should be of the form

$$\hat{v}_i(t) = f_i([d_{is}(t)]_{s\in\mathcal{S}_i}) \quad (6)$$

$$\omega_i(t) = g_i([d_{is}(t)]_{s\in\mathcal{S}_i}) \quad (7)$$

with certain functions $f_i : \mathbb{R}^m \to \mathbb{R}^2$ and $g_i : \mathbb{R}^m \to (-\pi, \pi]$. The controllers of the forms (6) and (7) are said to be *distributed* over 1D distance sensors.

Assume that we can assign the robots which can be detected by sensor $s \in \mathcal{S}_i$ through a function $h_i : \mathcal{S}_i \to \mathcal{V}$. Namely, $h_i(s) = j$ means that sensor s of robot $i \in \mathcal{V}\backslash\{1\}$ detects the distance from the surface of robot j. Let $\mathcal{N}_i \subset \mathcal{V}$ be the *detectable set* of robot i, namely the set of the robots which are detected by robot i as

$$\mathcal{N}_i = \{j \in \mathcal{V} : \exists s \in \mathcal{S}_i \text{ s.t. } h_i(s) = j\}. \quad (8)$$

Then, the detectable sets $\mathcal{N}_i$ for $i \in \mathcal{V}\backslash\{1\}$ define the network topology of the sensor detection.

2.2 Control Objective

The control objective is to achieve a desired formation, which is assigned as follows: For $i, j, k \in \mathcal{V}$ such that $i \neq j \neq k \neq i$, let $D^*_{ij} > 0$ be the desired distance between robots i and j and $\Theta^*_{jik} \in (-\pi, \pi]$ be the desired angle of the triangle with the vertexes of robots i, j and k at robot i. See Fig. 3 for the illustration. Then, achieving the desired formation is described as

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = D^*_{ij} \quad (9)$$

$$\lim_{t\to\infty} \angle(x_j(t) - x_i(t), x_k(t) - x_i(t)) = \Theta^*_{jik}, \quad (10)$$

which are the control objectives. Assume that these desired parameters satisfy

$$D^*_{ij} = D^*_{ji}, \ \Theta_{jik} = -\Theta_{kij} \quad (11)$$

$$D^*_{ij} \neq D^*_{ik} \text{ or } \Theta^*_{jik} \neq 0 \quad (12)$$

$$\Theta_{jik} + \Theta_{kji} + \Theta_{ikj} = \pi \quad (13)$$

$$(D^*_{ij})^2 + (D^*_{ik})^2 - 2D^*_{ij}D^*_{ik}\cos\Theta^*_{jik} = (D^*_{jk})^2. \quad (14)$$