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Stochastically Stable Equilibria for Evolutionary Snowdrift Games on Graphs ⋆

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Abstract: In this paper, we study two-player evolutionary snowdrift games on regular graphs and identify the stochastically stable equilibria for infinite populations. We consider four different update rules: birth-death(BD), death-birth(DB), imitation(IM) and pairwise comparison(PC). With the same values of cost and benefit of cooperation, we show that there is a unique stochastically stable equilibrium for evolutionary games on graphs. If the benefit-to-cost ratio is greater than 1.5, then the proportion of cooperators of a regular graph is higher than that of well-mixed population. And for BD and PC updating, the smaller graph degree can lead to more cooperators. Besides theoretical analysis, the results are also demonstrated by numerical simulations.

Keywords: Evolutionary Game, Stochastic Stability, Well-Mixed Population, Structured Population, Two-Player Game

1. INTRODUCTION

The evolutionary game theory was introduced in Sandholm (2010) and Skyrms (2014) as a theoretical analysis tool to study the strategic interactions of large populations of individuals over time. For evolutionary game theory, evolutionary stable strategies (ESS) become the focus of the study. According to Weibull (1997), an evolutionary stable strategy is a strategy adopted by the entire population, which can not be invaded by a minority of mutants playing a different strategy. In Taylor and Jonker (1978) and Schuster and Sigmund (1983), to model the evolutionary process, a set of differential equations, called replicator dynamics, are constructed to describe evolutionary game dynamics in the deterministic limit of an infinitely large, well-mixed population. Here, well-mixed means that all players are equally likely to interact with each other, in other words, the effects of spatial structures are ignored.

Up to now, there have been considerable efforts in investigating the evolutionary game dynamics over spatial structured networks. Ellison (1993) studies the coordination games in spatial systems and shows that localized interactions can facilitate faster convergence than global interactions. Then, Ohtsuki and Nowak (2006a), Nowak (2006) and Ohtsuki and Nowak (2008) consider the evolutionary game dynamics on cycles and other types of regular graphs. They find that the benefit-to-cost ratio of the altruistic act needs to exceed the number of neighbors per player for Prisoner's Dilemma. Meanwhile, Szabó and Fath (2007) give a tutorial-type overview of the evolutionary game theory for physicists. Fu et al. (2009) propose an efficient method to study all types of games on arbitrary graphs for weak selection. They formulate the game dynamics as a discrete Markov process by incorporating a detailed description of the microscopic dynamics. Tarnita et al. (2009) present a simple result that holds for a large variety of population structures. That is, the effect of population structure on strategy selection can be described by a single parameter. They present the values of the parameter for cases of well-mixed populations, the games on graphs, the games in phenotype space and the games on sets. Recently, Lanchier et al. (2015) investigate the evolutionary game on the lattice based on a continuous-time Markov chain.

Most of the above literature ignore the stochastic noises in the interacting processes. The limitation of ESS is that it treats each stochastic perturbation as an isolated event. In addition, the definition of ESS assumes that any small stochastic perturbation to the system will eventually die out. However, a system may continually be subjected to small stochastic noises that arise through mutations. Hence, the continuous small perturbation may affect the evolutionary process of games. To model the case with stochastic noises, the concept of stochastically stable strategies is exploited. The notion of stability for a stochastic dynamic system has been developed in Foster and Young (1990) with the insight that the stochastically stable set must be contained in the set corresponding to some minimum potential of the system. Then long-run equilibria are discussed in Kandori et al. (1993) and applied to symmetric coordinated games. Huang et al. (2015) proposes a stochastic model, which naturally combines frequency dependent selection and demographic fluctuations by assuming frequency dependent competition between different types in an individual-based model.

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The snowdrift game, sometimes referred to as a general class of noncooperative games in Basar and Olsder (1995), is particularly suitable for the study of the naturally generation mechanism of cooperation in large populations. Compared with the most celebrated game model of prisoner's dilemma, the fundamental feature in the snowdrift game is that, cooperation is the better option than defection when the opponent defects (Doebeli and Hauert, 2005). The main difference between them is that in the snowdrift game, a mixed strategy of cooperation and defection can emerge. The concept of the snowdrift game is presented by Smith and Price (1973), and it describes the scenario that if each player prefers not to cooperate with the other, then the non-collaboration of both players will lead to the worst possible case. Doebeli et al. (2004) consider the snowdrift game as a model for the evolution of cooperation. We have described clearly the condition under which there is a unique stochastically stable equilibrium in the multi-player snowdrift games for well-mixed populations in Liang et al. (2015).

Compared with Liang et al. (2015), which ignores the effects of spatial structure of the population, in this paper, we study the stochastic stability of evolutionary snowdrift games on regular graphs, where each node has exactly the same degree. Inspired by the works of deterministic evolutionary snowdrift games (Souza et al. (2009)) and spatial evolutionary games (Ohtsuki and Nowak (2006b)), we consider that stochastic effects and spatial effects may quantitatively change the asymptotic behavior of an evolutionary snowdrift game. Therefore, we focus on the stochastically stable equilibria for snowdrift games on regular graphs. We consider an infinitely large population, and the structure of the population is described by an infinite regular graph of degree k. Each player is represented by a node of the graph, and the edges denote who meets whom. We consider four different update rules: BD, DB, IM and PC. Compared with the most existing relevant results, the contributions of this paper are twofold: (i) we describe clearly the stochastically stable equilibria of evolutionary snowdrift games on regular graphs, while most existing literature considered deterministic evolutionary snowdrift games or stochastic games on well-mixed population; (ii) With the same values of cost and benefit of cooperation, we show that there is a unique stochastically stable equilibrium for evolutionary games on regular graphs. If the benefit-to-cost ratio is greater than 1.5, then the proportion of cooperators of a regular graph is higher than that of well-mixed population. We point out that it is possible to shift the stable equilibria of stochastic evolutionary snowdrift games by manipulating the degree of the graph and then control the proportion of cooperators in the structured populations.

The rest of the paper is organized as follows. In Section 2, we present the model of evolutionary dynamics of snowdrift games on regular graphs. In Section 3, we discuss the stochastic stability of well-mixed population games and structured population games. We give simulation examples in Section 4. Section 5 concludes this paper and suggests directions for future work.

2. PROBLEM FORMULATION

We consider evolutionary games in infinite well-mixed population and structured population. Each player has two strategies, to cooperate or defect, and at each time when it is matched up with another player. Let $x(t) \in [0,1]$ denote the proportion of cooperators in the whole population at time t, and correspondingly $1-x(t)$ denote the proportion of defectors.

Firstly, we recall two-player evolutionary games in well-mixed infinite population. Then the population dynamics of the cooperators over time can be described by the stochastic replicator dynamics (Sandholm, 2010)

$$dx(t) = x(t)(1-x(t))(f_C - f_D)dt + \sigma dw(t), \tag{1}$$

where f_C and f_D are the average fitness of cooperators and defectors, respectively, determined by the payoff matrices of the two-player games. In addition, the stochastic noise w is a standard Wiener process that captures mutations or other stochastic perturbations, and σ is the noise strength.

People usually visualize snowdrift games by considering drivers that are trapped in front of a huge snowdrift blocking their way home. The cooperating strategy, abbreviated by C, is then to shovel and the defecting strategy, abbreviated by D, is not to shovel. The benefit of clearing the snowdrift is denoted by b and the cost by c; naturally we assume $b > c > 0$. Then the normal form of the payoff matrix of the two-player snowdrift game (Souza et al., 2009) is

$$\begin{array}{ccc} & C & D \\ C & b-\frac{1}{2}c & b-c \\ D & b & 0 \end{array} \tag{2}$$

In the two-player game for well-mixed population, if both of the two players cooperate, then the cost is shared by the two and each of them receives the payoff $b - \frac{1}{2}c$; if one cooperates and the other defects, then the cooperator contributes to the cost completely and gets the payoff $b-c$ while the defector becomes a free-rider and obtains the biggest possible payoff b; and when neither of the two players cooperate, none of them gets any payoff. So when there are x portion of cooperators in the well-mixed population, the average fitness of a cooperator is

$$f_C = (b - \frac{1}{2}c)x + (b-c)(1-x),$$

and the average fitness of a defector is

$$f_D = bx.$$

Hence, the replicator dynamics (1) becomes

$$dx = x(1-x)\left(b - c - (b - \frac{c}{2})x\right)dt + \sigma dw. \tag{3}$$

Secondly, we study two-player evolutionary games on regular graphs, where each node has exactly the same degree. It is well known that every agent interacts with every other agent equally likely, in a well-mixed population. Whereas, real population is often not well-mixed, but arranged on a spatial graph. Hence, the players interact with their nearest neighbors, in spatial evolutionary games. Spatial