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Performance analysis of feedback loops with asynchronous sampling in the forward and return paths ⋆

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Abstract: Integral-quadratic constraint (IQC) based robust-stability analysis is used to establish a new linear matrix inequality (LMI) condition for verifying $\mathbf{L}_2$-gain performance for feedback interconnections of linear time-invariant continuous-time systems via asynchronously sampled signals in the forward and return paths. Applications of this condition in three simple numerical examples are compared to results for an existing LMI condition based on Lyapunov stability analysis. All three examples show that the IQC-based certificate can be much less conservative.

Keywords: sampled-data systems; robust performance analysis; networked control

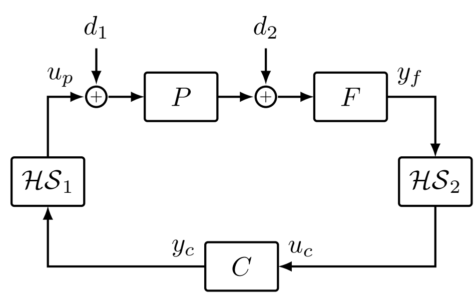

Fig. 1. The hybrid feedback interconnection of interest: P is a proper rational transfer function; F is a strictly-proper stable anti-aliasing transfer function; C is a strictly-proper rational transfer function; $\mathcal{HS}_i$ is the composition of an ideal sampling operator with uncertain sampling times and an ideal zero-order-hold operator.

1. INTRODUCTION

Kao and Cantoni (2015) develop an integral-quadratic constraint (IQC) based analysis approach to the verification of stability and $\mathbf{L}_2$-gain performance for feedback interconnections of the structure shown in Fig. 1. While the sampling is allowed to be aperiodic therein, the operators $\mathcal{S}_1$ and $\mathcal{S}_2$ are assumed to be synchronised. The extension to accommodate asynchrony in the forward and return paths, and generalisations of the set-up shown in Fig. 1 to accommodate delay or further processing in the communication of sampled signals, for example, are of relevance in the modelling of embedded and networked control systems. Indeed, the specific scope of this paper relates to the interaction of multiple continuous-time subsystems across a digital network via sampling.

In this paper, the work of Kao and Cantoni (2015) is extended in two ways: (i) it is shown that a simple modification can allow for the sampling in the forward and return paths to be asynchronous; and (ii) applications of the IQC-based performance certificate are empirically compared to results for an alternative that is based on Lyapunov analysis techniques (Suplin et al., 2007). For the three example systems considered, the numerical results show that the IQC-based certificate is substantially less conservative. For the sake of clarity and to facilitate comparison with alternative results, the communication of samples is taken to be direct here. Generalisations of this are part of ongoing work.

It is of note that the nature of the hybrid feedback interconnection studied in this paper is distinct from those typically seen within the context of sampled-data feedback control. Indeed, much recent work in this direction focuses on asynchronous static state-feedback, for example, (Seuret, 2012), (Briat and Seuret, 2012), (Fridman, 2010), and (Naghshtabrizi et al., 2008).

The remainder of the paper is organised as follows. The notation and basic mathematical preliminaries are presented in Section 2. The asynchronous sampled-data system is described in Section 3, and a linear-matrix-inequality (LMI) based $\mathbf{L}_2$-gain performance certificate is established by using IQC-based robust stability analysis techniques along the lines of Kao and Cantoni (2015). The applicability of an alternative Lyapunov stability analysis based LMI certificate from Suplin et al. (2007) is demonstrated in

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Section 4. The two certificates are compared for three example systems in Section 5.

2. NOTATION AND PRELIMINARIES

$\mathbb{R}$ ($\mathbb{C}$), $\mathbb{R}^n$ ($\mathbb{C}^n$), $\mathbb{R}^{p\times m}$ ($\mathbb{C}^{p\times m}$) denote the real (complex) numbers, n-dimensional real (complex) vectors, and $p\times m$ real (complex) matrices, respectively. Moreover, $\bar{\mathbb{R}} := \mathbb{R}\cup\{\pm\infty\}$. The column vector $(v_1, v_2, \ldots, v_n)^\top$ is constructed by stacking the vectors $v_1, v_2, \cdots, v_n$. Given $M \in \mathbb{C}^{p\times m}$, the transpose and conjugate transpose are denoted by $M^\top$ and M^*, respectively. When M is Hermitian (i.e., $M = M^*$), the notation $M \succ 0$ (resp., "$\succeq$", "$\prec$", "$\preceq$") denotes positive definite (resp., positive semi-definite, negative definite and negative semi-definite) in the sense $x^*Mx \geq cx^*x$ for some constant $c > 0$ and for all $x \in \mathbb{C}^n$.

2.1 *Signal spaces, operators and IQCs.*

Let $C^0[0,\infty)$ denote the space of continuous functions on $[0,\infty) \subset \mathbb{R}$ and $C^1[0,\infty)$ the space of continuously differentiable functions, on $[0,\infty) \subset \mathbb{R}$. Given a map $f : X \to Y$, the *restriction* of f to $A \subset X$ is the map $f|_A := ((x \in A \subset X) \mapsto (f(x) \in Y))$.

$\mathbf{L}_2^n$ denotes the space of Euclidean 2-norm square integrable functions $f : [0,\infty) \to \mathbb{R}^n$, with the usual norm and inner product denoted by $\|\cdot\|_{\mathbf{L}_2}$ and $\langle\cdot,\cdot\rangle_{\mathbf{L}_2}$, respectively. The superscripts for dimensions are dropped when the dimension is evident from the context. The extended $\mathbf{L}_2$ space is denoted by $\mathbf{L}_{2e}$. This consists of functions f that satisfy $P_T f \in \mathbf{L}_2$, where P_T denotes the truncation operator, defined as: $(P_T f)(t) = f(t)$ for $t \leq T$, and $(P_T f)(t) = 0$ otherwise. Note that $C^1[0,\infty) \subset C^0[0,\infty) \subset \mathbf{L}_{2e}$.

Given, a measurable and uniformly bounded multiplier $\Pi = (\omega \in \bar{\mathbb{R}} \mapsto \Pi(j\omega) \in \mathbb{C}^{(m+p)\times(m+p)})$ such that $\Pi(j\omega) = \Pi(j\omega)^*$, it is said that a bounded operator $\Delta : \mathbf{L}_2^m \to \mathbf{L}_2^p$ satisfies the integral quadratic constraint defined by Π if

$$\sigma_\Pi(v, \Delta v) := \int_{-\infty}^{\infty} \begin{bmatrix} \hat{v}(j\omega) \\ \widehat{\Delta v} \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{v}(j\omega) \\ \widehat{\Delta v} \end{bmatrix} \mathrm{d}\omega \geq 0 \quad (1)$$

holds for all $v \in \mathbf{L}_2$, where $\hat{\cdot}$ denotes the Fourier transform. If Π is dependent on a parameter X, then $\Pi(X)$ denotes this dependence.

2.2 *Sampling and zero-order-hold operators.*

For $i = 1, 2$ and given sampling sequence $\{t_{i,k}\}_{k=0}^\infty$ satisfying $t_{i,0} = 0$, $t_{i,(k+1)} > t_{i,k}$, and $\lim_{k\to\infty} t_{i,k} = \infty$, the ideal sampling operator $\mathcal{S}_i$ maps a continuous-time signal $v \in C^0[0,\infty) \cap \mathbf{L}_{2e}$ to a sequence $\{\tilde{v}_k\}_{k=0}^\infty$, where $\tilde{v}_k = v(t_{i,k})$. The time-varying sampling interval is given by $h_{i,k} := t_{i,(k+1)} - t_{i,k}$, and is assumed to satisfy $0 < h_{i,k} < \bar{h}_i$, where $\bar{h}_i$ is a finite upper-bound on the maximum inter-sample time. The ideal zero-order-hold operator $\mathcal{H}_i$ maps a discrete-time sequence $\{\tilde{v}_k\}_{k=0}^\infty$ to a continuous-time signal $v \in \mathbf{L}_{2e}$, where $v(t) = \tilde{v}_k$ for $t_{i,k} \leq t < t_{i,(k+1)}$ for $k = 1, 2, \ldots$; i.e., there is direct undelayed and untransformed communication between $\mathcal{S}_i$ and $\mathcal{H}_i$ (generalisation of this is the subject of ongoing work). Correspondingly, the sample-and-hold operator $\mathcal{HS}_i := \mathcal{H}_i \circ \mathcal{S}_i : C^0[0,\infty) \cap \mathbf{L}_{2e} \to \mathbf{L}_{2e}$ satisfies

$$(\mathcal{H}_i\mathcal{S}_i)(v))(t) = v(t_{i,k}) \text{ for } t \in [t_{i,k}, t_{i,(k+1)}),\ k = 0, 1, \ldots.$$

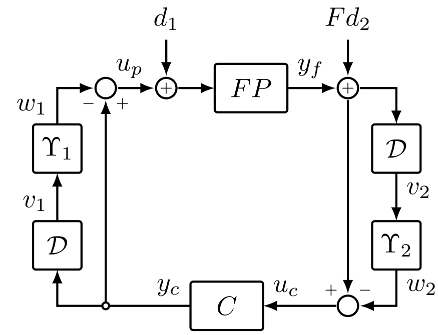

Fig. 2. Loop transformation which recovers the original continuous time feedback interconnection.

Finally, let $\Upsilon_i : \mathbf{L}_{2e} \to \mathbf{L}_{2e}$ be defined by

$$(\Upsilon_i v)(t) := \int_{t_{i,k}}^{t} v(\tau)\,\mathrm{d}\tau, \quad (2)$$

for $t \in [t_{i,k}, t_{i,(k+1)})$, $k = 0, 1, \ldots$. Note, that $\Upsilon_i = (1 - \mathcal{HS}_i) \circ \mathcal{I}$, where $\mathcal{I}$ denotes the integral operator defined by $(\mathcal{I}v)(t) = \int_0^t v(\tau)\,\mathrm{d}\tau$.

3. ROBUST STABILITY AND PERFORMANCE OF ASYNCHRONOUS SAMPLED-DATA SYSTEMS

In this section a finite-dimensional LMI certificate is established for verifying the stability and performance of the asynchronous sampled-data system shown in Fig. 1. The formulation closely follows (Kao and Cantoni, 2015).

The following properties are assumed to hold for the system of interest in Fig. 1: *(i)* P has a proper rational transfer function; *(ii)* F has a strictly-proper stable anti-aliasing transfer function; and *(iii)* C has a strictly-proper rational transfer function (i.e., the anti-aliasing filter for $\mathcal{S}_1$ has been absorbed into C); *(iv)* C internally stabilises FP.

Consider the system in Fig. 1. Since C and F are both strictly-proper rational transfer functions, the signals y_c and y_f (i.e., inputs to $\mathcal{HS}_1$ and $\mathcal{HS}_2$, respectively) are continuously differentiable. As such, the operators $\mathcal{HS}_i$ can instead be replaced with the restriction to $C^1[0,\infty)$. Now, the identity

$$\mathcal{HS}_i|_{C^1[0,\infty)} = 1 - (1 - \mathcal{HS}_i) \circ \mathcal{I} \circ \mathcal{D} = 1 - \Upsilon_i \circ \mathcal{D},$$

where $\mathcal{D}$ denotes the differentiation operator such that $(\mathcal{I} \circ \mathcal{D})v = v$ for $v \in C^1[0,\infty)$, and a loop-transformation of Fig. 1, yield the system shown in Fig. 2. In this transformed system the continuous-time feedback interconnection $[FP, C]$ is additively perturbed by the effects of the asynchronous sample-and-hold operators. This system can be represented as the following structured feedback interconnection:

$$[G, \Upsilon_1 \oplus \Upsilon_2] := \begin{cases} v = G\tilde{w} + e_1 \\ \tilde{w} = (\Upsilon_1 \oplus \Upsilon_2)v + e_2 \end{cases}, \quad (3)$$

where $v = (v_1, v_2)^\top$, $e_1 = (e_{11}, e_{12})^\top$, $e_2 = (e_{21}, e_{22})^\top$, and G is the operator with the stable transfer function