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Unilateral Control Structure Under Communication Rate Constraint

H. Okajima * **Y. Minami** ** **N. Matsunaga** ***

* *Graduate School of Science and Technology, Kumamoto University, 2-39-1 Kurokami,Kumamoto-City 860-8555 Japan (e-mail: okajima@cs.kumamoto-u.ac.jp).*
** *Graduate School of Information Science, Nara Institute of Science and Technology 8916-5 Takayama, Ikoma, Nara 630-0192 Japan (e-mail: minami@is.naist.jp)*
*** *Graduate School of Science and Technology, Kumamoto University, 2-39-1 Kurokami,Kumamoto-City 860-8555 Japan (e-mail: matunaga@cs.kumamoto-u.ac.jp).*

Abstract: This study proposes a design method of unilateral control systems under communication rate constraints. In case that it is required to control under a communication rate constraint, the input signal should be minimized in the meaning of the data rate by signal quantization. Feedback-type dynamic quantizers are effective for signal quantization. A design method of the dynamic quantizer under communication rate constraints has been proposed by our previous research. A unilateral control structure is proposed for minimizing the effect of the quantization error in this paper. The design method of the quantizer is applied for the proposed structure. Effectiveness of the proposed system with the designed quantizer is assessed through numerical examples.

Keywords: Communication rate constraint, Networked control systems, Quantization, Dynamic quantizer, Unilateral systems

1. INTRODUCTION

The networked control systems (NCSs) have been recently studied (Antsaklis et al. (2004); Nair et al. (2007); Tsumura et al. (2009)). Control signals should be compressed appropriately by using quantizers to satisfy limited communication rate (Wong et al. (1997, 1999); Tatikonda et al. (2004); Brockett et al. (2000); Picasso et al. (2007)). There exist performance degradations caused by the quantization because plants are controlled by compressed (quantized) signals. Feedback type dynamic quantizers are known to play an important role of minimizing performance degradation (Inose et al. (1962); Quevedo et al. (2004); Azuma et al. (2008.2,.); Minami et al. (2007); Azuma et al. (2012); Norsworthy et al. (1996)). These quantizers are consisting of a filter and a static quantizer, they use previous quantization error values to generate quantizer output. Such quantization methods are widely used in signal processing (Inose et al. (1962); Norsworthy et al. (1996)) such as in AD-DA conversion systems and data compressor for music audio signals. In recent years, feedback type dynamic quantization methods have been addressed in control engineering fields (Azuma et al. (2008.2,.); Minami et al. (2007); Azuma et al. (2012); Quevedo et al. (2004)). Performance degradation decreases if an appropriate filter is chosen in the dynamic quantizer. An optimal dynamic quantizer based on an ℓ_∞ cost function has been proposed in (Azuma et al. (2008.2)). This optimal quantizer was expressed analytically as a function of plant parameters. Moreover, many researches such as feedback control (Azuma et al. (2008.9)), the non-minimum-phase plants (Minami et al. (2007)) and non-linear systems (Azuma et al. (2012)) have been proposed for control systems.

The authors have extended the researches about dynamic quantizers to the communication rate constraint problems. When we want to use the dynamic quantizers for NCS, the output level number of dynamic quantizers should be satisfied the limited communication rate.

In this paper, we propose a control structure for unilateral control systems to overcome the effect by the communication rate constraint. By using the proposed structure, the effect of quantization noise can be reduced with the equipped dynamic quantizers. Effectiveness of the proposed structure is illustrated by numerical examples.

In the remainder of the manuscript, a set of $n \times m$ real matrices is denoted as $\mathcal{R}^{n\times m}$. $\mathcal{R}_+$ is the set of positive real numbers and I is the identity matrix. For a matrix H, H^T and $\rho(H)$ correspond its transpose and spectral radius, respectively. For a vector $X = \{x_1, x_2, \cdots, x_k, \cdots\}$, $\|X\|$ represents its infinity norm. Consequently, $\|X\| = \sup_k \|x_k\|$ holds.

2. PROBLEM FORMULATION

2.1 Control systems with a communication channel

A single input single output discrete-time plant P is defined as

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$$P\begin{cases} x_p(k+1) &= A_p x_p(k) + B_p u_p(k), \\ y_p(k) &= C_p x_p(k), \end{cases} \tag{1}$$

where $x_p \in R^{n_p\times 1}$ is the state, $u_p \in R$ is the control input, $y_p \in R$ is the control output. $A_p \in R^{n_p\times n_p}$, $B_p \in R^{n_p\times 1}$ and $C_p \in R^{1\times n_p}$ are constant matrices of P, and $x_p(0)$ is the initial state. $x_p(0) = 0$ is assumed.

A desired control system is shown in Fig. 1. Figure 1 includes a feedforward controller C_1 and a feedback controller C_2. The controllers $C_i(i = 1, 2)$ are defined as the following state space realization:

$$C_i\begin{cases} x_{ci}(k+1) &= A_{ci} x_{ci}(k) + B_{ci} u_{ci}(k), \\ y_{ci}(k) &= C_{ci} x_{ci}(k) + D_{ci} u_{ci}(k), \end{cases} \tag{2}$$

where $x_{ci} \in R^{n_{ci}\times 1}$ are the state, $u_{ci} \in R$ are the input of C_i, $y_{ci} \in R$ are the output of C_i. $A_{ci} \in R^{n_{ci}\times n_{ci}}$, $B_{ci} \in R^{n_{ci}\times 1}$, $D_{ci} \in R^{1\times 1}$ and $C_{ci} \in R^{1\times n_{ci}}$ are given constant matrices of C_i, and $x_{ci}(0)$ is the initial state. We assume C_1 is stable, and C_2 stabilize the system in Fig. 1.

Figure 2 shows the structure of a control system equipped with a communication channel, in which u is an outer signal and y is the plant output of (1), respectively. K_P and K_R are controllers and Q is a quantizer. K_P, K_R and Q are design parameters in this paper.

Signal u may be regarded as an operating signal or a command, such as a telesurgery operation. The quantizer Q transforms the high-resolution outer signal u into a lower resolution signal u_q. No delay or bit missing is assumed to occur. K_P, K_R and Q should be designed to minimize the difference between the desired system in Fig. 1 and the system in Fig. 2 in terms of input-output relation.

The number of quantization levels N depends on the communication rate of the channel. When M [bits] data are transmitted through the channel over a sampling period of the control system, N should satisfy the following inequality.

$$N \le 2^M \tag{3}$$

Although, N is assumed to be even in this study.

The outer signal u is constrained by upper and lower boundaries, giving the signal range $U = [u_{\min}, u_{\max}]$. Hence, the outer signal u is assumed to satisfy the following relation.

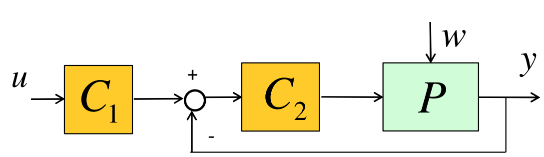

Fig. 1. Desired control system

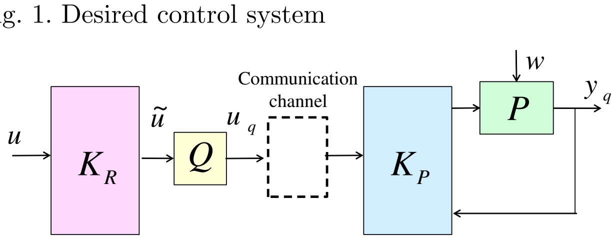

Fig. 2. Unilateral control system with quantizer Q

$$u(k) \in U, \ \forall k \tag{4}$$

2.2 Dynamic quantizer form

A feedback-type dynamic quantizer is defined as follows.

$$Q\begin{cases} \xi(k+1) &= A\xi(k) - Bu(k) + Bu_q(k), \\ u_q(k) &= Q_{st}\left[C\xi(k) + u(k)\right], \end{cases} . \tag{5}$$

where Q_{st} is a uniform static quantizer with saturation. $A \in R^{n_q\times n_q}$, $B \in R^{n_q\times 1}$ and $C \in R^{1\times n_q}$ are constant matrices of the feedback type dynamic quantizer Q. The initial state is assumed as $\xi(0) = 0$. The number of the quantization levels are given as N in Q_{st}. Q_{st} is defined using a quantization interval $d \in \mathcal{R}_+$ and a center point $h \in \mathcal{R}$. Its level interval is the same for input and output axes. Figure 3 shows an example of Q_{st} (Solid line, $N = 8$, Mid-riser type uniform static quantizer). Center of Q_{st} is (h, h).

2.3 Control objective

In this paper, we attempt to design K_R, K_P and Q such that y_q in Fig. 2 approximates y in Fig. 1. To this end, we evaluate the error signal $e = y - y_q$, and consider the following evaluation function:

$$E(Q) = \sup_{u(k)\in U} \|\mathcal{Y} - \mathcal{Y}_q\|, \tag{6}$$

where $\mathcal{Y} = \{y(1), y(2), \cdots\}$ and $\mathcal{Y}_q = \{y_q(1), y_q(2), \cdots\}$ are the output time series. Because $E(Q)$ produces the maximum value for $e(k)$, y is expected to be similar to y_r in case $E(Q)$ becomes smaller. In existing dynamic quantizer designs (Azuma et al. (2008.2,.); Minami et al. (2007); Azuma et al. (2012)), $E(Q)$ is used as a performance index for these quantizers.

The objective of this research is to design filters K_R, K_P and quantizer Q based on the performance index (6).

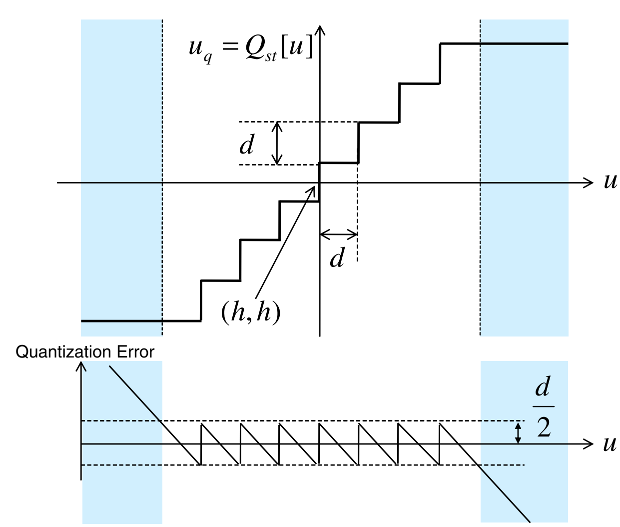

Fig. 3. Static quantizer Q_{st} including saturation