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Self-triggered control for communication reduction in networked systems

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Abstract: We study networked control of linear discrete-time systems using self-triggered strategies to reduce the amount of communication. At each transmission, the controller determines the next transmission time in advance based on the current state. We propose three self-triggered strategies which guarantee control performance based on a quadratic cost function. They have different characteristics with respect to the computation load for finding the transmission times. Through a numerical example, we demonstrate the tradeoff between computation loads and transmission frequencies.

Keywords: Networked control systems, Self-triggered control, Hybrid systems

1. INTRODUCTION

In recent years, the use of communication networks in control systems has drastically increased for connecting plants with controllers which may be remotely located. Due to the shared nature of networks as well as the limited computation in embedded devices, it is important to design such networked control systems with certain considerations to keep the communication and computation loads low. In this respect, conventional digital control techniques employing periodic sampling may not be ideal.

Reduction in communication can be achieved by activating transmissions only when it is necessary. This is the underlying idea in the strategies of event-triggered control and self-triggered control, which have lately gained much attention; see, e.g., (Mazo and Tabuada (2008); Heemels et al. (2013); Cetinkaya et al. (2016)) and the references therein. In event-triggered control, the state of the plant is continuously monitored, but only when the state value has sufficiently changed and satisfies certain conditions, communication is triggered for the controller to be updated (Tabuada (2007)). On the other hand, in self-triggered control, when the controller transmits the new control input to the plant, it is accompanied with the information regarding the time when the sensor should send the state the next time (Wang and Lemmon (2009); Gommans et al. (2014)). Since the transmission times are determined in advance, self-triggered control may require more communication in general compared to the event-triggered case. The advantage is however that monitoring of the state is unnecessary at the sensor side.

In this paper, we study self-triggered control strategies for linear time-invariant systems in the discrete-time domain (Eqtami et al. (2010); Brunner et al. (2015); Gommans et al. (2014); Zhang et al. (2015)). We develop three self-triggered schemes that require different levels of real-time computation for finding the next transmission time at the controller. While all of them are guaranteed to achieve given control performance, they exhibit tradeoffs between the necessary computation and the length of waiting times before the next transmissions. Hence, depending on the system requirements, the most appropriate option should be chosen. The first strategy is based on computing the future state using the plant model and is hence more computationally intensive. The second strategy requires much less real-time computation by using bounds on the state trajectories, but in general is more demanding in terms of communication. In the third one, the amount of on-line computation is further reduced by partitioning the state space into a finite number of regions, where each region has the corresponding transmission times.

In all three strategies, we follow the control method developed by (Ishii and Francis (2002)) in the context of quantized control. There, a Lyapunov-based approach is developed for finding the so-called dwell time in continuous time, which is in fact closely related to event-triggered control. These references further consider quantization of the control input and how to reduce the data rate in communication. However, in this paper, we employ only the ideas for the sampling part of the results there. The interesting aspect is that the state space is projected on a two-dimensional space, which enables us to especially reduce real-time computation in the off-line case.

This paper is organized as follows. In Section 2, we formulate the networked control problem studied. In Section 3, we present a Lyapunov-based sufficient condition to guarantee the desired level of control performance. Sections 4, 5, and 6 provide the details of the three self-triggered control strategies. We illustrate the results through a numerical example in Section 7. In Section 8, we give some concluding remarks.

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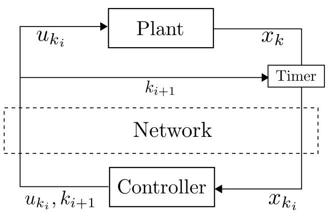

Fig. 1. Networked control system

2. PROBLEM FORMULATION

Consider the networked control system depicted in Fig. 1. Here, the plant is a linear time-invariant system given by

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

where the state and the control input at time $k \in \mathbb{Z}_+$ are denoted, respectively, by $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}$. Assume the matrix A to be unstable and the pair (A, B) controllable.

This networked system can be described as follows: The sensor and the actuator on the plant side communicate with the remote controller over a network, which is free from latencies and packet losses. The objective is to reduce the number of transmissions over the network by means of self-triggered control. Hence, though the sensor measures the state at every time step $k \in \mathbb{Z}_+$, it is sent to the controller only at transmission times denoted by $k_i \in \mathbb{Z}_+$, $i \in \mathbb{Z}_+$, with $k_0 = 0$ and $k_i < k_{i+1}$. At time k_i, the controller broadcasts the control input together with the next transmission time k_{i+1} to the actuator and the sensor. The sensor will send the next measurement at time k_{i+1} while the same input is applied until then, that is,

$$u_k = u_{k_i} \text{ for } k = k_i, k_i + 1, \ldots, k_{i+1} - 1.$$

We employ the quadratic cost function given as

$$J(x_0) := \sum_{k=0}^{\infty}(x_k' Q x_k + Ru_k^2). \tag{2}$$

where the weight matrix $Q \in \mathbb{R}^{n\times n}$ is positive definite and $R > 0$. It is well known that with the state feedback $u_k = -Kx_k$, we achieve the optimal cost of $J_{\mathrm{opt}}(x_0) := x_0' P x_0$, where P is the unique positive-definite solution to the discrete-time algebraic Ricatti equation

$$A'PA - P - A'PB(B'PB + R)^{-1}B'PA + Q = 0, \tag{3}$$

and the optimal feedback gain is $K := B'PA/(R+B'PB)$.

With self-triggered control, we can reduce communication over channels, but in turn we must relax the performance constraint. So, for a given $\epsilon > 0$, we design self-triggered control schemes to achieve the performance bound

$$J(x_0) \le (1 + \epsilon)J_{\mathrm{opt}}(x_0). \tag{4}$$

Hence, the problem of this paper is formulated as follows: For the networked control system in Fig. 1, design self-triggered control schemes that compute the next transmission time k_{i+1} at time k_i based on the information of past states and inputs available at the controller and achieve quadratic stability and the performance constraint (4) for the closed-loop system.

In this work, we propose three self-triggered control schemes, where their difference lies in the necessary computational resources at the controller. Specifically, we will see that with more computation at the controller for computing the transmission times, better performance can be attained with respect to control and communication.

3. A LYAPUNOV-BASED CONDITION

We derive a condition for the update times k_i, which forms the basis for the proposed self-triggered control schemes.

Let $V(x) := x'Px$ be the Lyapunov-like function and let $\Delta V(x, u) := V(Ax + Bu) - V(x)$. The lemma below provides a condition that will be used in the three self-triggered schemes.

Lemma 1. The networked control system in Fig. 1 is quadratically stable and satisfies (4) if

$$(R + B'PB)(u_k + Kx_k)^2 \le -\epsilon\Delta V(x_k, u_k),\ \forall k \in \mathbb{Z}_+. \tag{5}$$

Next, we project the condition (5) on a two-dimensional space by using the technique in (Ishii and Francis (2002)); see also (Kang and Ishii (2015)) for a related approach.

It follows from (3) and the definition of $\Delta V(x_k, u_k)$ that

$$\begin{aligned}\Delta V(x_k, u_k) &= V(x_{k+1}) - V(x_k)\\ &= x_k'(A'PA - P)x_k + 2x_k'A'PBu_k + B'PBu_k^2\\ &= -x_k'\big[Q - K'(B'PB + R)K\big]x_k\\ &\quad + 2u_k(B'PB + R)Kx_k + B'PBu_k^2\\ &= (B'PB + R)\Big[-x_k'\Big(\frac{Q}{B'PB + R} - K'K\Big)x_k\\ &\qquad + 2u_kKx_k + u_k^2\Big] - Ru_k^2.\end{aligned}$$

To simplify the expression, we introduce the coordinate transformation $\overline{x}_k := Ux_k$ with $U := \big(Q/(B'PB+R)\big)^{1/2}$, and also let $\overline{K} = KU^{-1}$. Then, we have

$$\begin{aligned}\Delta V(x_k, u_k) = (B'PB + R)\big(&-\overline{x}_k'\overline{x}_k + \overline{x}_k'\overline{K}'\overline{K}\overline{x}_k\\ &+ 2u_k\overline{K}\overline{x}_k + u_k^2\big) - Ru_k^2.\end{aligned} \tag{6}$$

Here, we further define $\mathcal{M} := (\ker \overline{K})^{\perp} = \mathrm{Im}\, \overline{K}'$ and $e := \overline{K}'/\|\overline{K}\|$. We observe that, by this representation, for any $\overline{x} \in \mathbb{R}^n$, there exists a unique pair $\alpha \in \mathbb{R}$ and $y \in \mathcal{M}^{\perp}$ such that $\overline{x} = \alpha e + y$.

For $u \in \mathbb{R}$, denote by $\mathcal{X}(u)$ the set of states satisfying (5):

$$\mathcal{X}(u) = \big\{\overline{x} \in \mathbb{R}^n : (R + B'PB)(u + \overline{K}\overline{x})^2 \le -\epsilon\Delta V\,(\overline{x}, u)\,\big\}.$$

Lemma 2. The set $\mathcal{X}(u)$ can be expressed as

$$\mathcal{X}(u) = \Big\{\alpha e + y\colon\ \alpha \in \mathbb{R},\ y \in \mathcal{M}^{\perp},\ g(\alpha, u)^2 \le \|y\|^2\Big\},$$

where

$$\begin{aligned}g(\alpha, u) := \Big\{\Big[\Big(1 + \frac{1}{\epsilon}\Big)\|\overline{K}\|^2 - 1\Big]\alpha^2 + 2\Big(1 + \frac{1}{\epsilon}\Big)u\,\|\overline{K}\|\,\alpha\\ + \Big(1 + \frac{1}{\epsilon} - \frac{R}{R + B'PB}\Big)u^2\Big\}^{1/2}.\end{aligned} \tag{7}$$

Proof: From (5) and (6), $x \in \mathcal{X}(u)$ implies

$$\begin{aligned}(R + B'PB)(u + \overline{K}\overline{x})^2 \le -\epsilon(B'PB + R)(-\overline{x}'\overline{x} + \overline{x}'\overline{K}'\overline{K}\overline{x}\\ + 2u\overline{K}\overline{x} + u^2) + \epsilon Ru^2.\end{aligned} \tag{8}$$

By using the representation $\overline{x} = \alpha e + y$, we obtain $\overline{K}\,\overline{x} = \|\overline{K}\|\,\alpha$ and $\overline{x}'\overline{x} = \alpha^2 + \|y\|^2$. Thus,