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Kalman Filtering with Unknown Sensor Measurement Losses ⋆

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Abstract: This work studies the state estimation problem of a networked linear system where a sensor and an estimator are connected via a lossy network. If the measurement loss is *known* to the estimator, the minimum variance estimate is easily computed by the intermittent Kalman filter (IKF). However, this does not hold for the case of *unknown* measurement losses, and we have to address the non-Gaussianity/non-linearity of the networked system. By exploiting the measurement loss process and the IKF, we design three recursive suboptimal filters for state estimation, i.e., BKF-I, BKF-II and RBPF. The BKF-I is based on the MAP estimator of the loss process and the BKF-II is derived by an estimate of the conditional loss probability. The RBPF is an effective sequential importance sampling algorithm by marginalizing out the loss process. A target tracking example is included to illustrate their effectiveness and shows the tradeoff between computation complexity and estimation accuracy of the proposed filters.

Keywords: Linear systems, intermittent Kalman filter, sensor measurement losses, particle filter.

1. INTRODUCTION

The research on the state estimation with stochastic measurement losses bears a vast body of literature, see e.g. Wu et al. (2014); You et al. (2015) and references therein. However, most of the existing works assume that the measurement loss process is known to the estimator and focus on the stability of the intermittent Kalman filter (IKF), which is originally proposed in Sinopoli et al. (2004) as the minimum variance estimate. For instance, Sinopoli et al. (2004) shows the existence of a critical measurement loss probability beyond which the IKF may be unstable, otherwise always convergent. In You et al. (2011), this critical value for certain types of systems are explicitly obtained under the Markovian measurement losses. In real applications, the estimator may not able to discern whether the sensor measurement is lost during the transmission in the noisy channel as the received data can be pure noise or the noise-corrupted sensor measurement. Thus, it is of importance to study the state estimation problem under *unknown* sensor measurement losses.

Clearly, the lack of the measurement loss information results in a non-Gaussian/non-linear networked system where the IKF is no longer applicable. While there are some generic estimation methods for non-linear/non-Gaussian systems (Van Der Merwe et al., 2000), e.g., extended KF, unscented KF and particle filter (PF), they do not particularly explore the feature of the current problem. To this end, we model the measurement loss process by a binary sequence $\{\gamma_k\}$, i.e., $\gamma_k = 1$ means that the sensor measurement is received and $\gamma_k = 0$ indicates the loss of the sensor measurement. Our objective is to design recursive suboptimal filters to address these issues. Motivated by the optimality of the IKF, a natural idea is that we can first estimate γ_k under the maximum posterior criterion, based on which a Bayesian Kalman filter I (BKF-I) is derived. Clearly, it is easy to understand the intuition and to implement this algorithm. In addition, we derive a Bayesian Kalman filter II (BKF-II) by estimating the conditional loss probability, which is a compromise between the standard Kalman filter and the IKF. Both algorithms reduce to the IKF if the measurement loss process $\{\gamma_k\}$ is known to the estimator.

Another method to address the non-gaussianity/non-linearity lies in the use of particles to approximate the conditional density. However, the amount of computations required for the PF in high-dimensional state space is extremely large. To increase the sampling efficiency, one can marginalize out some of the states and use standard algorithms such as the Kalman filter to estimate them. Then, the PF is applied to estimate the rest of states, which is called Rao-Blackwellised particle Filter (RBPF). The implementation and comparison between standard PF and this method are well documented in Doucet et al. (2000); Van Der Merwe et al. (2000); Gustafsson et al. (2002). In this work, we adopt this idea and use the PF to estimate the conditional distribution of the measurement loss process $\{\gamma_k\}$, which is binary valued and requires only a few number of particles to approximate its distribution. Then, the state of the networked system is estimated by the IKF. Finally, a target tracking example is included to illustrate the effectiveness of the BKF-I, BKF-II and the RBPF. It is interesting that there exists a tradeoff between computation complexity and estimation performance of the proposed filters.

The rest of this paper is organized as follows. In Section 2, we formulate the estimation problem. In Section 3, we design the BKF-I and the BKF-II based on the Bayes' theorem and the Kalman filter. In Section 4, we derive the RBPF to deal with the networked estimation problem. Simulation is performed in Sections 5 to compare the performance of the above three filters. Finally, we draw the conclusions in Section 6.

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2. PROBLEM FORMULATION

Consider a discrete-time linear system where the measurements are transmitted to a remote estimator via a noisy channel:

$$x_{k+1} = Ax_k + w_k$$
$$y_k = \gamma_k \cdot Cx_k + v_k$$

where $x_k \in \mathbb{R}^n$ and $y_k \in \mathbb{R}^m$ are the vector states and measurements. $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are independent white Gaussian noises with zero means and covariance matrices $Q \geq 0$ and $R \geq 0$, respectively. γ_k is a binary random variable and represents the sensor measurement loss process. In particular, $\gamma_k = 1$ indicates the sensor measurement is contained in arrival data y_k while $\gamma_k = 0$ means that the estimator only receive pure noise. Moreover, the system (A, Q, C) is stabilizable and detectable. The initial state x_0 is a random Gaussian vector with mean $\bar{x}_0$ and covariance matrix $\Sigma_0 > 0$.

Different from Sinopoli et al. (2004), the binary process $\{\gamma_k\}$ in the new scenario is *unknown* to the estimator, which renders the IKF inapplicable. However, it is still very helpful in designing an effective filter in the current situation. To elaborate it, define $\Gamma_k = \{\gamma_0, ..., \gamma_k\}$ and $Y_k = \{y_0, ..., y_k\}$, and the conditional minimum variance estimate and error covariance matrices are given by

$$\begin{aligned}
\hat{x}_{k|k-1} &= \mathbb{E}[x_k|Y_{k-1}, \Gamma_{k-1}] \\
\hat{x}_{k|k} &= \mathbb{E}[x_k|Y_k, \Gamma_k] \\
\Sigma_{k|k-1} &= \mathbb{E}[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^{\mathrm{T}}|Y_{k-1}, \Gamma_{k-1}] \\
\Sigma_{k|k} &= \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^{\mathrm{T}}|Y_k, \Gamma_k] \\
\hat{y}_{k|k-1} &= \mathbb{E}[y_k|Y_{k-1}, \Gamma_{k-1}].
\end{aligned} \tag{1}$$

Then, the measurement update of the IKF in Sinopoli et al. (2004) is given by

$$\begin{aligned}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + \gamma_k K_k (y_k - C\hat{x}_{k|k-1}) \\
\Sigma_{k|k} &= \Sigma_{k|k-1} - \gamma_k K_k C \Sigma_{k|k-1}
\end{aligned} \tag{2}$$

and the time update is the same as the KF, i.e.,

$$\begin{aligned}
\hat{x}_{k+1|k} &= A\hat{x}_{k|k} \\
\Sigma_{k+1|k} &= A\Sigma_{k|k}A^{\mathrm{T}} + Q,
\end{aligned} \tag{3}$$

where the Kalman gain $K_k = \Sigma_{k|k-1}C^{\mathrm{T}}(C\Sigma_{k|k-1}C^{\mathrm{T}} + R)^{-1}$ and $\hat{x}_{0|-1} = \bar{x}_0, \Sigma_{0|-1} = \Sigma_0$.

In this work, we consider a more general and realistic situation where γ_k is unknown to the estimator. Then, x_k conditioned on Y_k is not Gaussian, which is significantly different from the IKF. Finding an effective minimum variance filter for this networked system is very difficult. Furthermore, such an optimal filter is expected to be too complex in real-time applications. To derive recursive estimate with unknown measurement losses, we propose three recursive suboptimal filters in the sequel.

3. BAYESIAN KALMAN FILTERS

3.1 Bayesian Kalman Filter I

We design a non-linear filter called Bayesian Kalman Filter I (BKF-I) to recursively compute the state estimate of the networked linear system with unknown measurement losses. To this end, an intuitive idea is that we first estimate the measurement losses Γ_k, based on which the IKF (2) is then applied to compute the state estimate. Since γ_k takes only binary values, it is natural to adopt the maximum a posteriori probability (MAP) estimator, i.e., the MAP estimator of Γ_k is given as follows:

$$\hat{\Gamma}_k = \text{argmax}_{\Gamma_k} p(\Gamma_k|Y_k)$$

where $\hat{\Gamma}_k = \{\hat{\gamma}_0, ..., \hat{\gamma}_k\}$. Then, substitute $\hat{\Gamma}_k$ into (2), we obtain the BKF-I. Thus, the remaining problem reduces to the estimation of Γ_k.

Moreover, it follows from the Bayes' formulas that

$$\begin{aligned}
p(\Gamma_k|Y_k) &= p(\gamma_k, \Gamma_{k-1}|Y_k) \\
&= \frac{p(\gamma_k, \Gamma_{k-1}, y_k|Y_{k-1})p(Y_{k-1})}{p(Y_k)}.
\end{aligned}$$

To recursively compute it, we notice that

$$\begin{aligned}
&p(\gamma_k, \Gamma_{k-1}, y_k|Y_{k-1}) \\
&= p(y_k|\gamma_k, \Gamma_{k-1}, Y_{k-1})p(\gamma_k, \Gamma_{k-1}|Y_{k-1}) \\
&= p(y_k|\gamma_k, \Gamma_{k-1}, Y_{k-1})p(\gamma_k|\Gamma_{k-1}, Y_{k-1})p(\Gamma_{k-1}|Y_{k-1}).
\end{aligned}$$

Combining the above, it is clear that

$$\begin{aligned}
&p(\Gamma_k|Y_k) = p(y_k|Y_{k-1}) \times \\
&\quad p(y_k|\gamma_k, \Gamma_{k-1}, Y_{k-1})p(\gamma_k|\Gamma_{k-1}, Y_{k-1})p(\Gamma_{k-1}|Y_{k-1}).
\end{aligned} \tag{4}$$

To compute $p(\Gamma_k|Y_k)$, we thus have to consider all possible values of Y_k and Γ_k, which grow unboundedly. Unless that γ_k is an independent process, it is generically impossible to estimate Γ_k recursively even if we know $p(\Gamma_{k-1}|Y_{k-1})$. Clearly, in the application problems like positioning and target tracking, it is preferable to device recursive algorithms. Thus, we consider to approximately compute $p(\Gamma_k|Y_k)$.

A natural idea is to use $\hat{\Gamma}_{k-1}$ rather than all possible values of Γ_{k-1} to estimate γ_k. By substituting Γ_{k-1} with $\hat{\Gamma}_{k-1}$, it follows from (4) that

$$\begin{aligned}
&p(\Gamma_k|Y_k) \approx p(\gamma_k, \hat{\Gamma}_{k-1}|Y_k) \\
&\propto p(y_k|\gamma_k, \hat{\Gamma}_{k-1}, Y_{k-1})p(\gamma_k|\hat{\Gamma}_{k-1}, Y_{k-1})p(\hat{\Gamma}_{k-1}|Y_{k-1}).
\end{aligned}$$

Since $\hat{\Gamma}_{k-1}$ is known, our objective becomes finding γ_k to approximately maximize the posterior probability. As γ_k is a binary variable, it follows that

$$\begin{aligned}
\hat{\gamma}_k &= \text{argmax}_{\gamma_k} p(\gamma_k, \hat{\Gamma}_{k-1}|Y_k) \\
&= \begin{cases} 1, p(\gamma_k = 1, \hat{\Gamma}_{k-1}|Y_k) > p(\gamma_k = 0, \hat{\Gamma}_{k-1}|Y_k) \\ 0, \text{otherwise} \end{cases}
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
&\frac{p(\gamma_k = 1, \hat{\Gamma}_{k-1}|Y_k)}{p(\gamma_k = 0, \hat{\Gamma}_{k-1}|Y_k)} = \\
&\frac{p(y_k|\gamma_k = 1, \hat{\Gamma}_{k-1}, Y_{k-1})p(\gamma_k = 1|\hat{\Gamma}_{k-1}, Y_{k-1})}{p(y_k|\gamma_k = 0, \hat{\Gamma}_{k-1}, Y_{k-1})p(\gamma_k = 0|\hat{\Gamma}_{k-1}, Y_{k-1})}.
\end{aligned} \tag{6}$$

In addition, we denote the probability density function of the Gaussian distribution with mean μ and covariance matrix σ^2 by $N(\mu, \sigma^2)$. Once Γ_k is known, it follows from the IKF that

$$\begin{aligned}
&p(y_k|\gamma_k, Y_{k-1}, \hat{\Gamma}_{k-1}) \\
&= \begin{cases} N(C\hat{x}_{k|k-1}, C\Sigma_{k|k-1}C^{\mathrm{T}} + R), & \gamma_k = 1 \\ N(0, R), & \gamma_k = 0 \end{cases}
\end{aligned}$$

where $\hat{x}_{k|k-1}$ and $\Sigma_{k|k-1}$ are computed by (2) with Γ_{k-1} being replaced by $\hat{\Gamma}_{k-1}$. With the prior probability distribution of γ_k, we are able to compute $p(\gamma_k|Y_{k-1}, \hat{\Gamma}_{k-1})$. Two common cases are illustrated below.